Introduction to Game Theory and Applications

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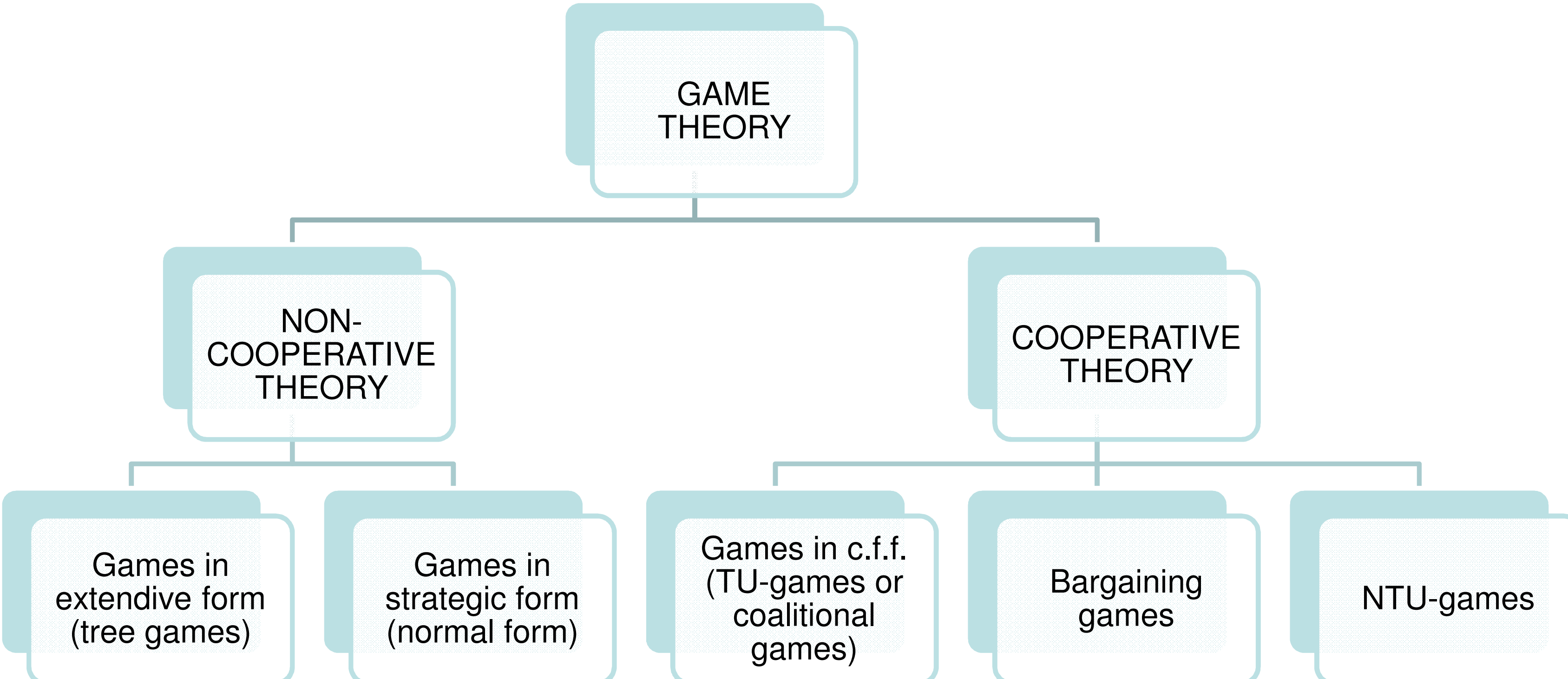


- Dominant strategies
- Nash eq. (NE)
- Subgame perfect NE
- NE & refinements

…

- Core
- Shapley value
- Nucleolus
- τ-value
- PMAS

….

- Nash sol.
- Kalai-Smorodinsky

….

- CORE
- NTU-value
- Compromise value

…

No binding agreements
No side payments
Q: Optimal behaviour in conflict situations

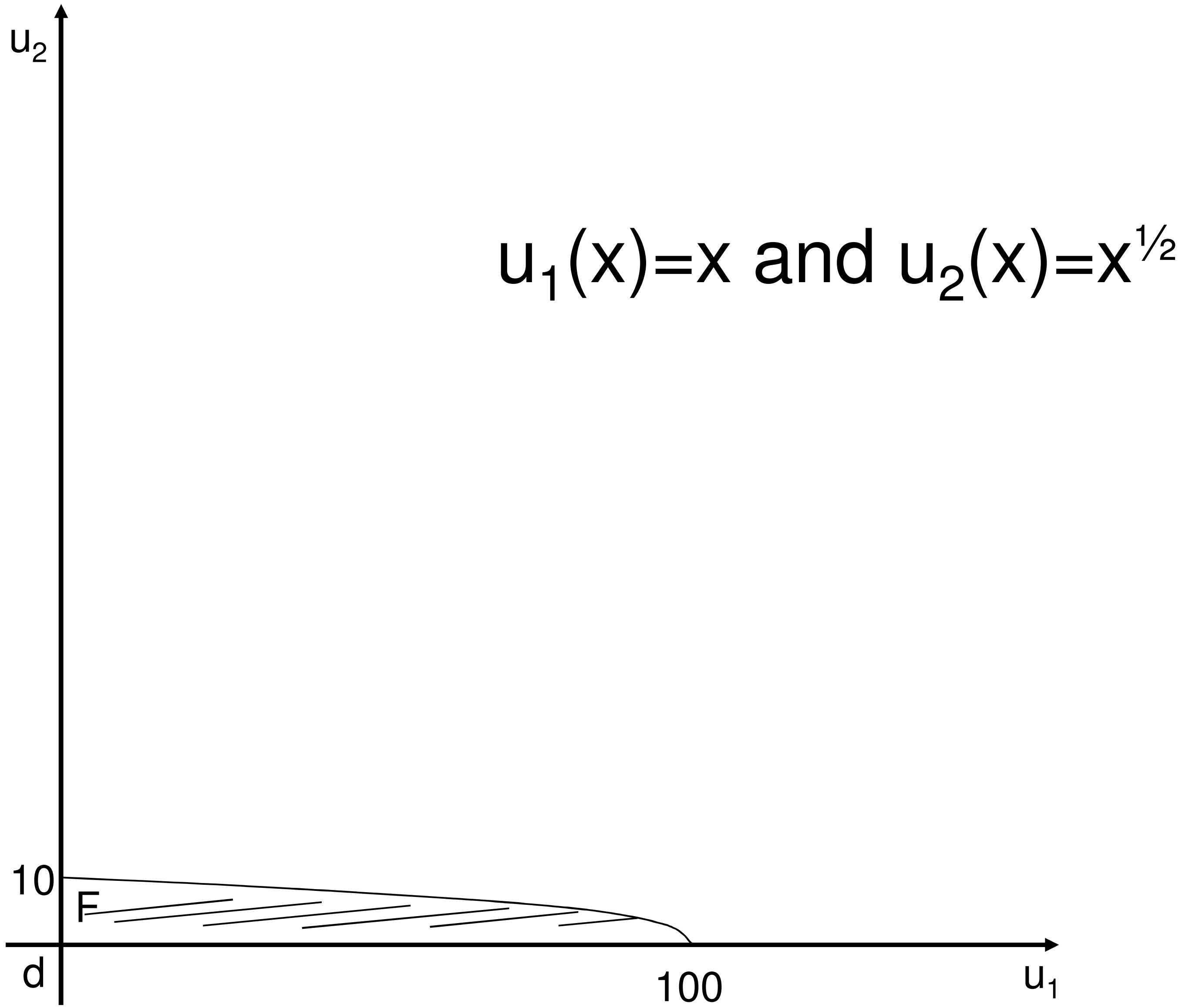
binding agreements
side payments are possible (sometimes)
Q: Reasonable (cost, reward)-sharing

Bargaining

- There are 100 euros on the table and two persons (players?) must search an agreement on how to divide them.
- If they do not find an agreement, money stays on the table.
- Each division is possible, even those in which some money remain on the table.
- **Q**: How to model this interactive situation?

Strong assumptions are the price for simple models

- agents utilities (satisfying vNM conditions) are common knowledge
  - Strong assumption: imagine if you have to buy a house… (bargaining under incomplete information: see Myerson (1979))
- u_1,u_2:IR→IR are two utilities functions (vNM) which describe the preferences of the agents.
  - In the specific case assume $u_1(x)=x$ and $u_2(x)=x^{½}$


$u_1(x)=x$ and $u_2(x)=x^{1/2}$
u_2
10
F
d
100
u_1

Bargaining games

- A two persons *bargaining game* is an ordered pair (F,d), where
  - F is a subset of IR^2
  - d is a point in IR^2
- The elements of F are said *feasible outcomes* (utility pairs) which the player can reach if they cooperate
- In case of no cooperation the *disagreement outcome* (or *status quo*) d results, with utility d_i for player $i \in \{1,2\}$
- **Q**: on which outcome to agree?

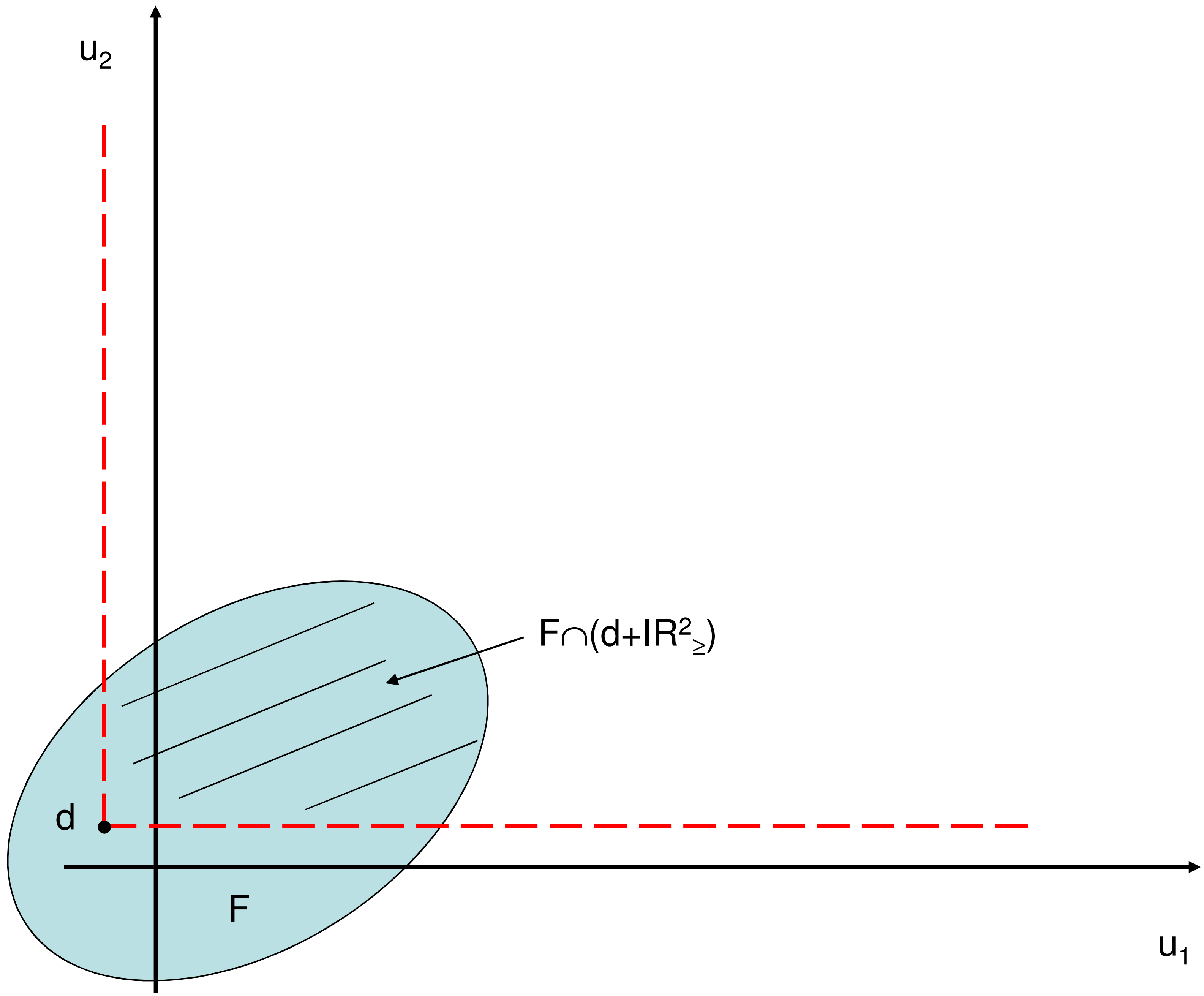
Further assumptions

B is the set of all bargaining games (F,d) such that:

- p.1) $F \subseteq IR^2$ is convex and closed
- p.2) $d \in F$
- p.3) $F \cap \{(u_1,u_2) \in IR^2 | \, u_1 \geq d_1, \, u_2 \geq d_2\}$ is bounded.

Definitions:

- A bargaining game is *essential* if there exists an element $(u_1,u_2) \in F$ such that $u_1 > d_1$, $u_2 > d_2$
- A *solution* Φ:**B**→ IR^2 is a map which assign to each bargaining game (F,d) (with properties 1,2, and 3) an utility vector in IR^2
- Later we will denote $(d+IR^2_{\geq}) = \{(d_1+x_1, \, d_2+x_2) | (x_1, \, x_2) \in IR^2_{\geq}\}$


u2
u1
d
F
F∩(d+IR²≥)

Nash axiomatic approach

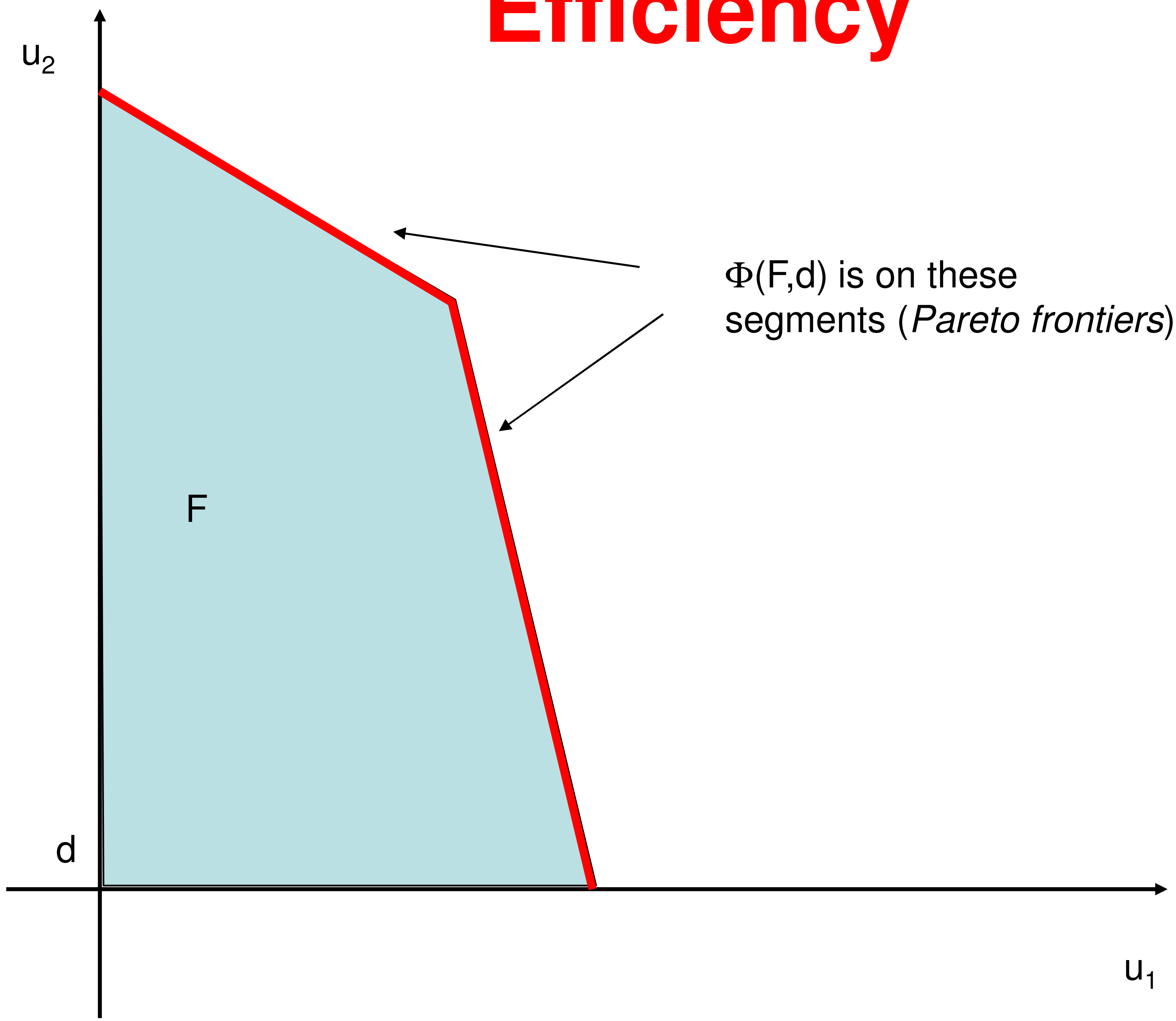
- Nash did not define an “a priori” Φ, but gave “reasonable” properties that each solution Φ should satisfy.

Prop. 1 Efficiency (EFF)

$\Phi(F,d) \in F$ and it is a Pareto optimal point,

i.e. there is no element $u \in F$ with $u \neq \Phi(F,d)$ such that $u_1 \geq \Phi_1(F,d)$ and $u_2 \geq \Phi_2(F,d)$.

Efficiency



Nash axiomatic approach (2)

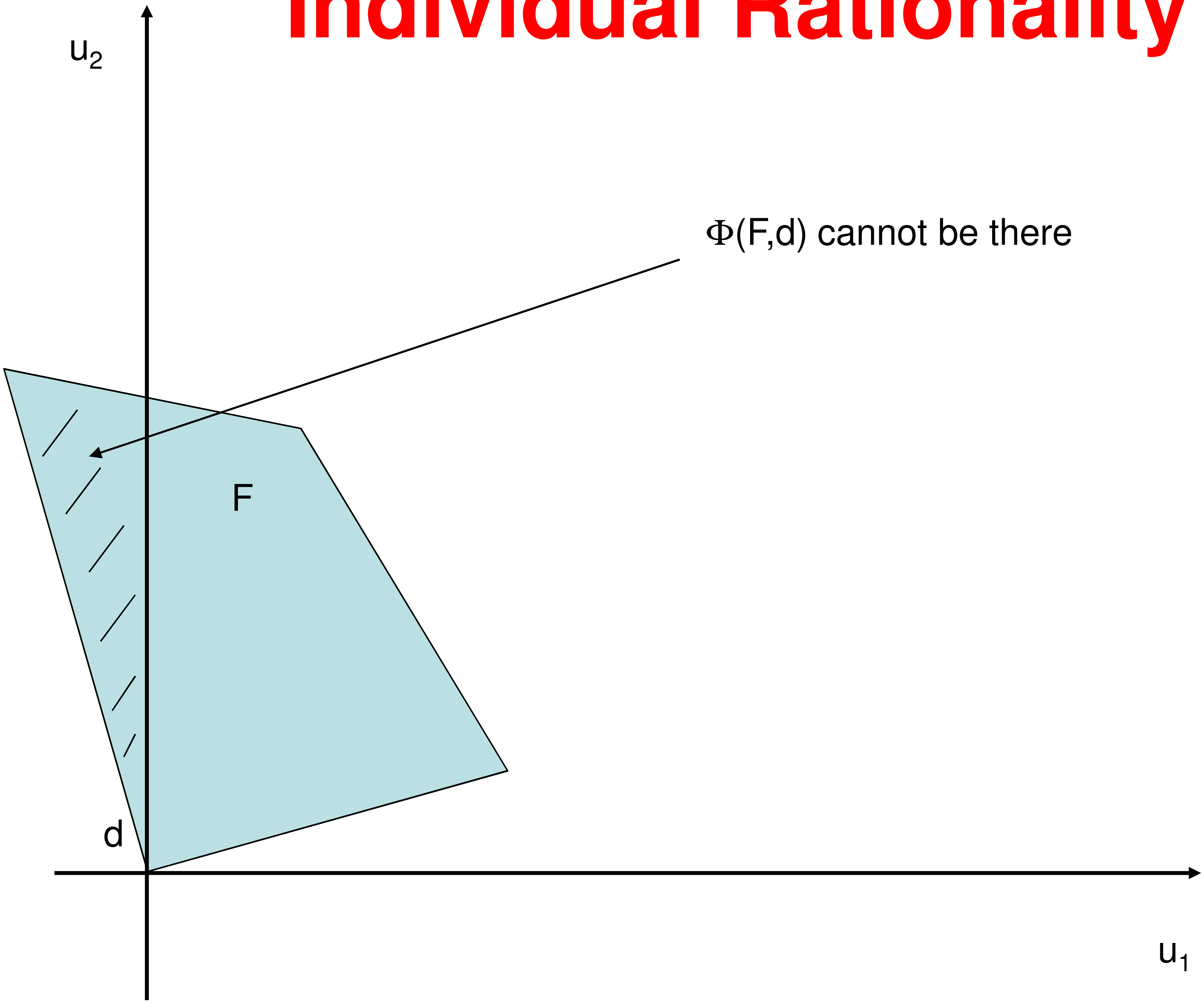
Prop. 2 Individual Rationality (INDR)

$\Phi_1(F,d) \geq d_1$ and $\Phi_2(F,d) \geq d_2$

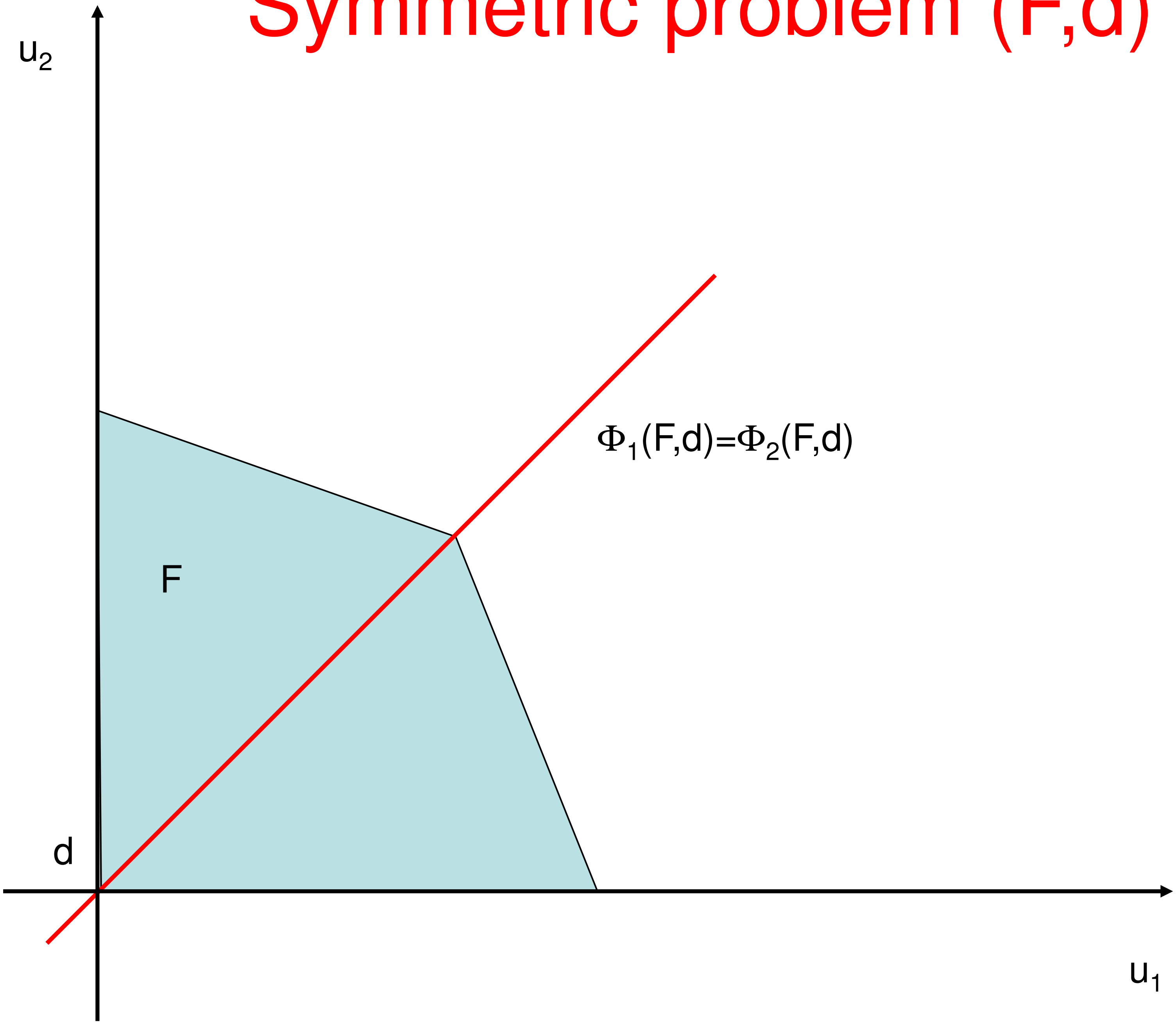
Prop. 3 Symmetry (SYM)

$$\left.\begin{array}{l} d_1 = d_2 \\ \\ (u_1,u_2) \in F \Leftrightarrow (u_2,u_1) \in F \end{array}\right\} \Rightarrow \Phi_1(F,d) = \Phi_2(F,d)$$

Individual Rationality



Symmetric problem (F,d)



Nash axiomatic approach (3)

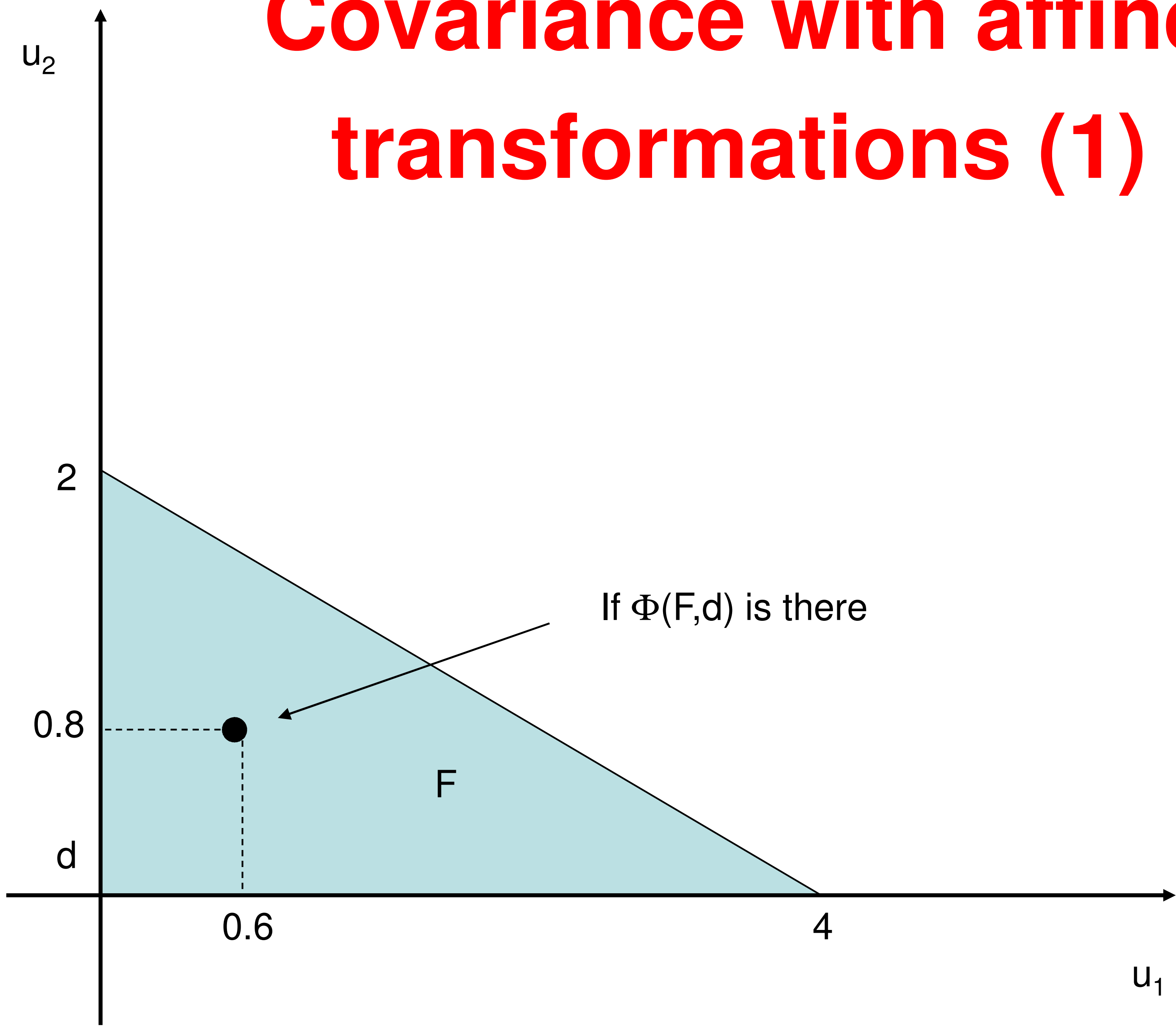
Prop. 4 Covariance with affine transformations (COV)

For each $\lambda_1, \lambda_2, \gamma_1, \gamma_2 \in \mathrm{IR}$ s.t. $\lambda_1, \lambda_2 > 0$ define:

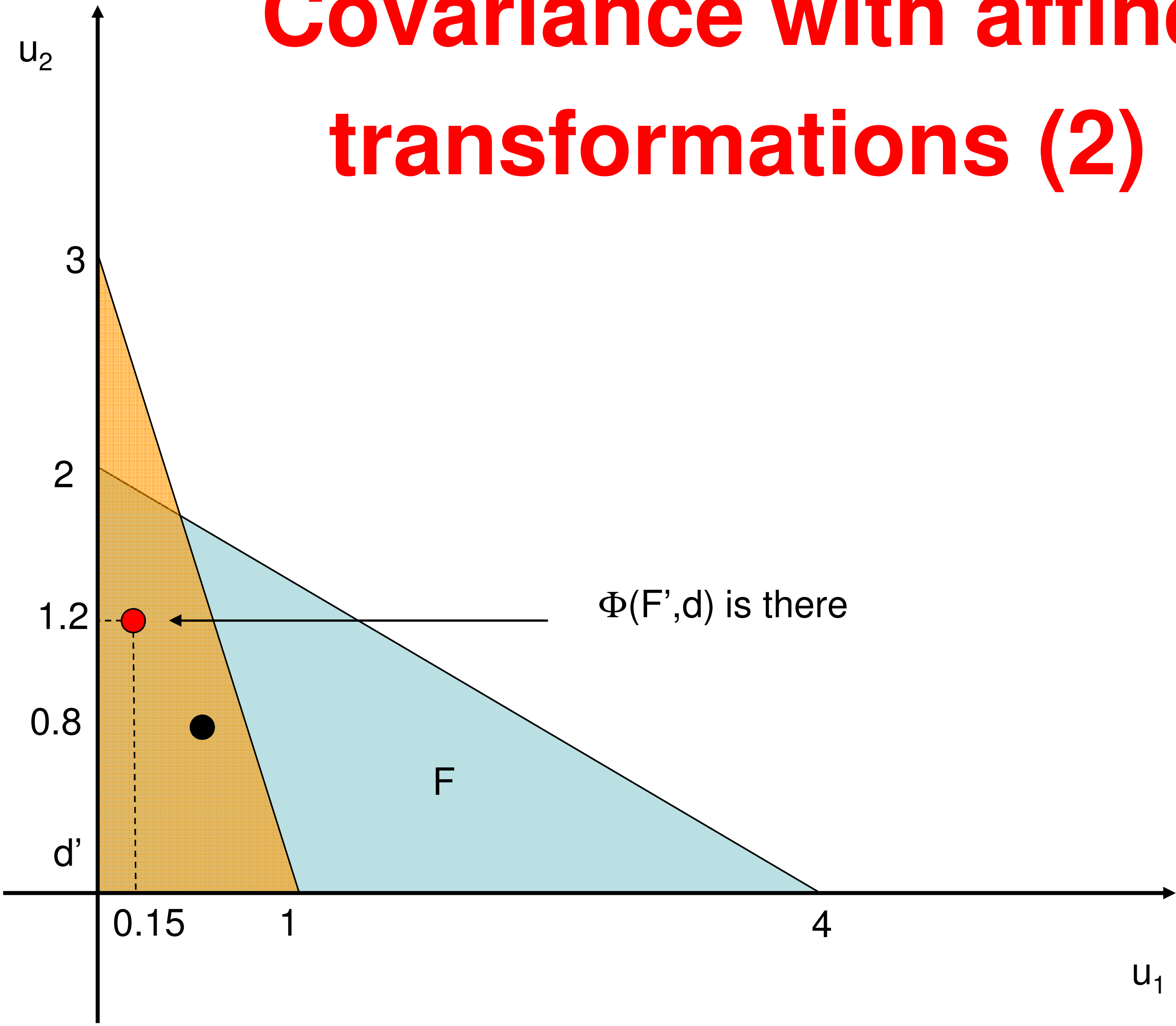
- $F' = \{(\lambda_1 u_1 + \gamma_1, \lambda_2 u_2 + \gamma_2) : (u_1, u_2) \in F\}$ and
- $d' = (\lambda_1 d_1 + \gamma_1, \lambda_2 d_2 + \gamma_2)$

Then $\Phi(F', d') = (\lambda_1 \Phi_1(F', d') + \gamma_1, \lambda_2 \Phi_2(F', d') + \gamma_2)$

Covariance with affine transformations (1)



Covariance with affine transformations (2)



Nash approach (3)

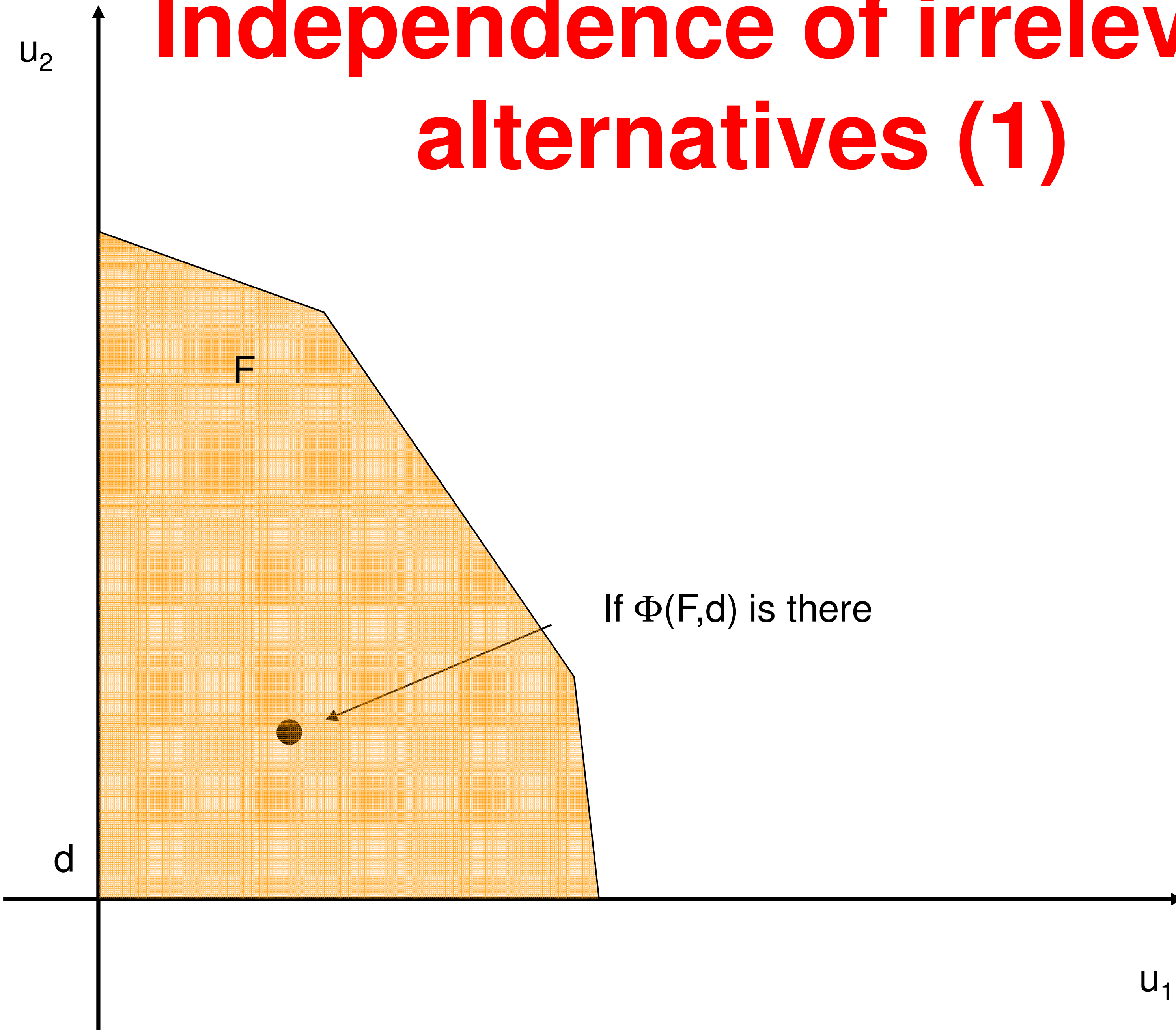
Prop. 5 Independence of irrelevant alternatives (IIA)

Let $(F,d),(F',d) \in \mathbf{B}$ be such that $F' \subseteq F$. If $\Phi(F,d) \in F'$,

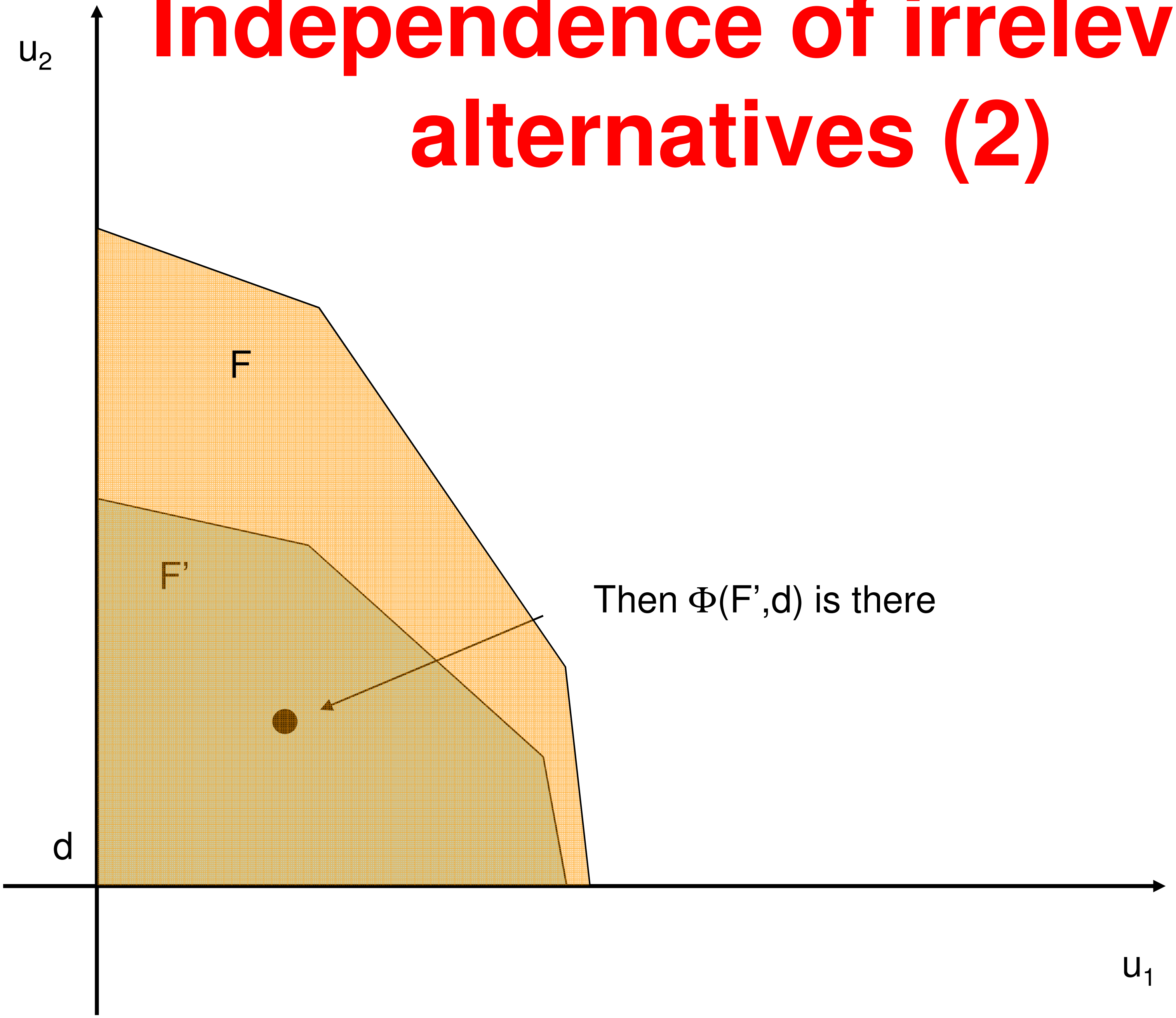
then $\Phi(F,d) = \Phi(F',d)$.

Note: $(F',d) \in \mathbf{B} \Rightarrow d \in F'$

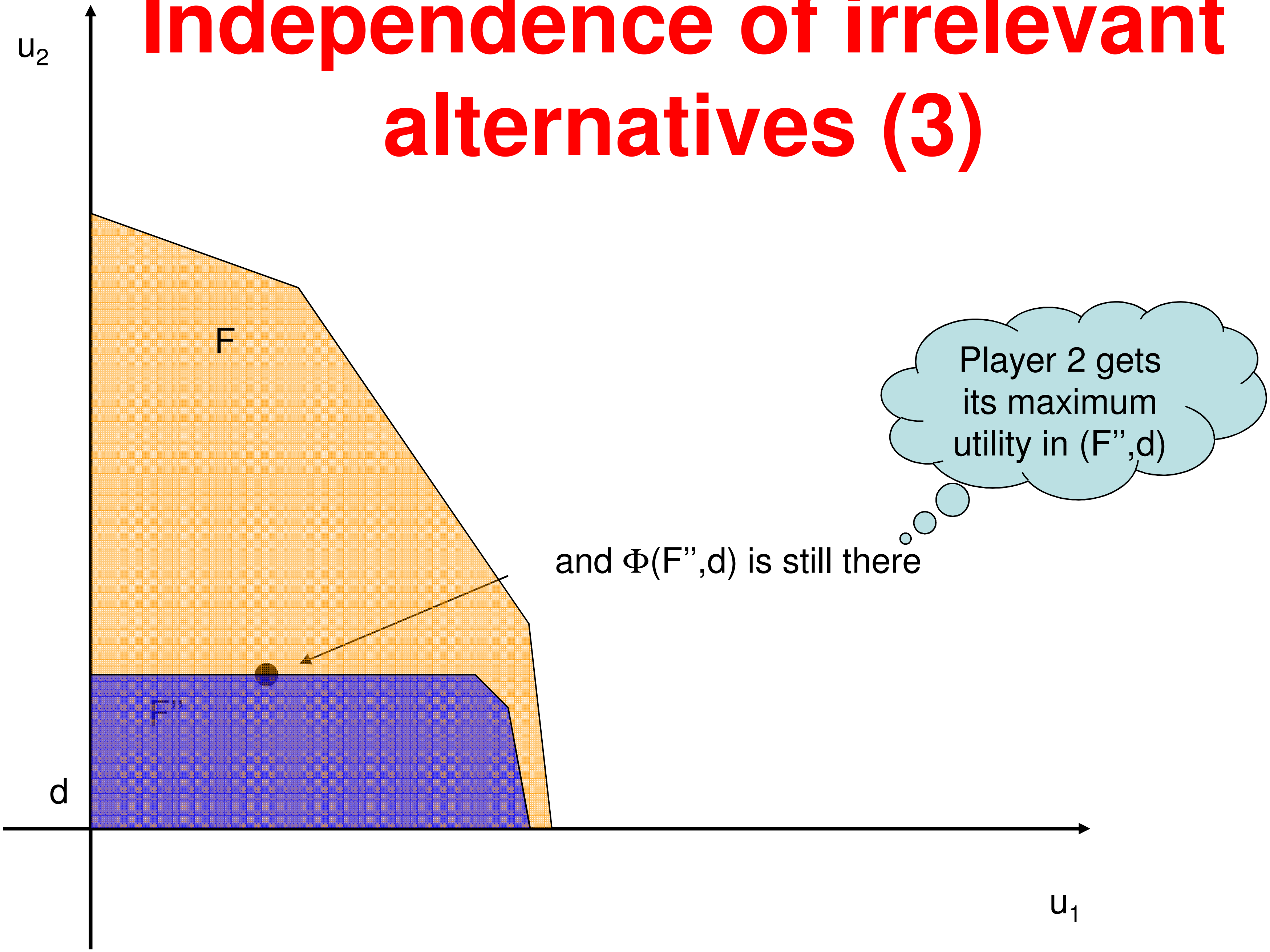
Independence of irrelevant alternatives (1)



Independence of irrelevant alternatives (2)



Independence of irrelevant alternatives (3)



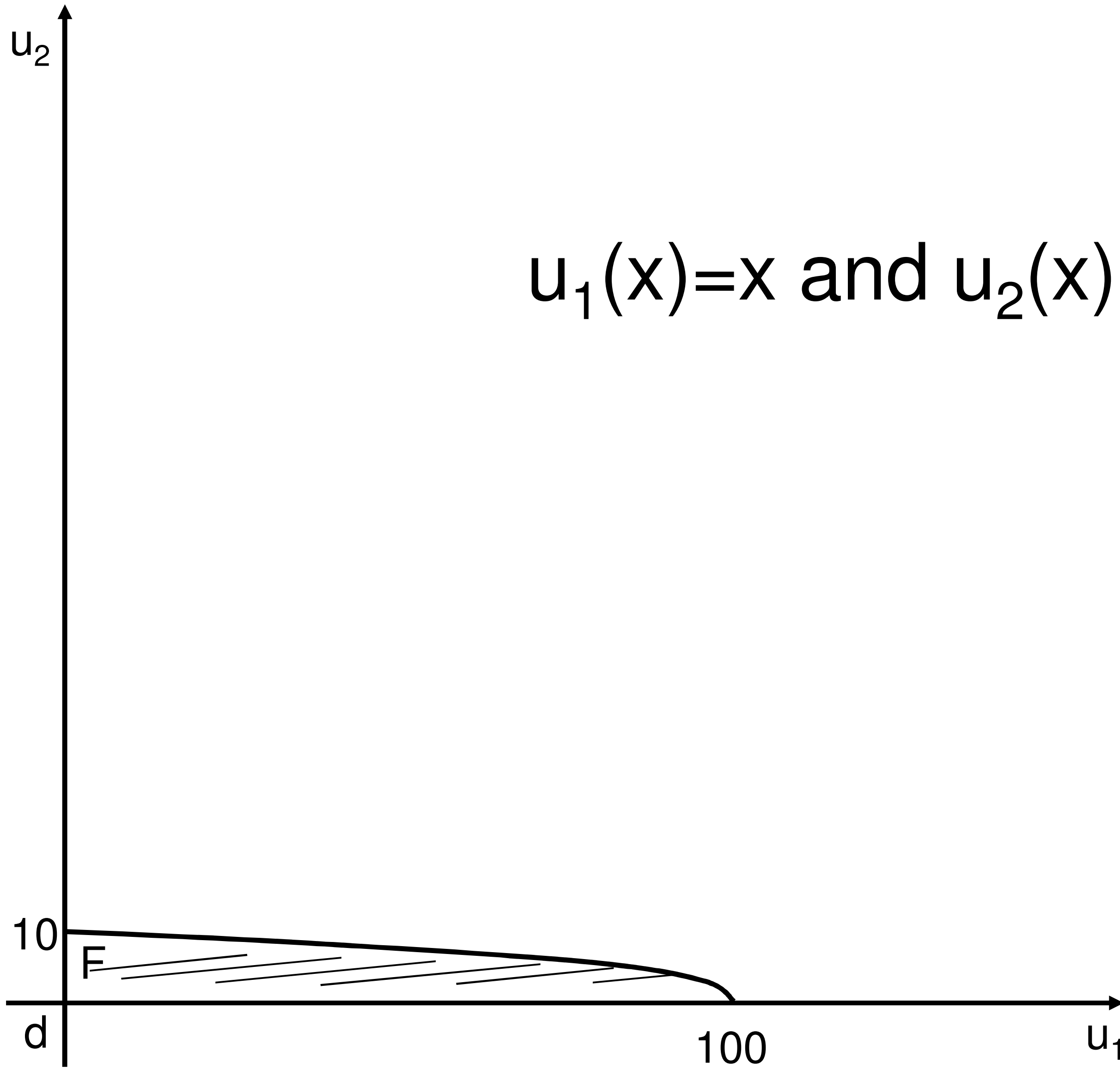
Nash solution (1950)

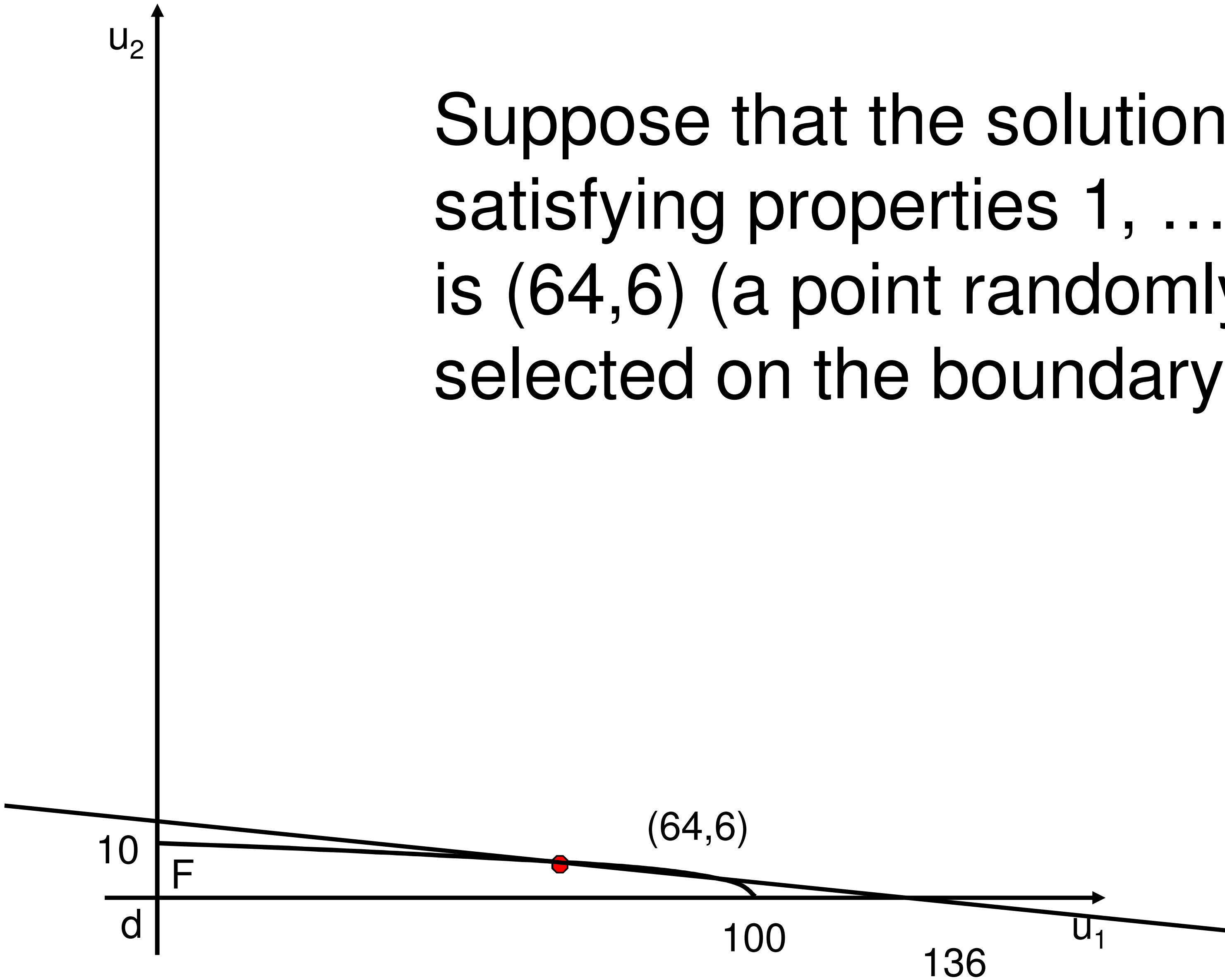
Theorem

There exists one and only one solution Φ defined on **B** which satisfies properties EFF, INDR, SYM, COV and IIA. Moreover, if $(F,d)\in$ **B** is essential, we have that:

$$\Phi(F,d)=\text{argmax}\{(u_1-d_1)(u_2-d_2) \mid (u_1,u_2)\in F\cap(d+ \mathrm{IR}^2_{\geq})\} \qquad (*)$$

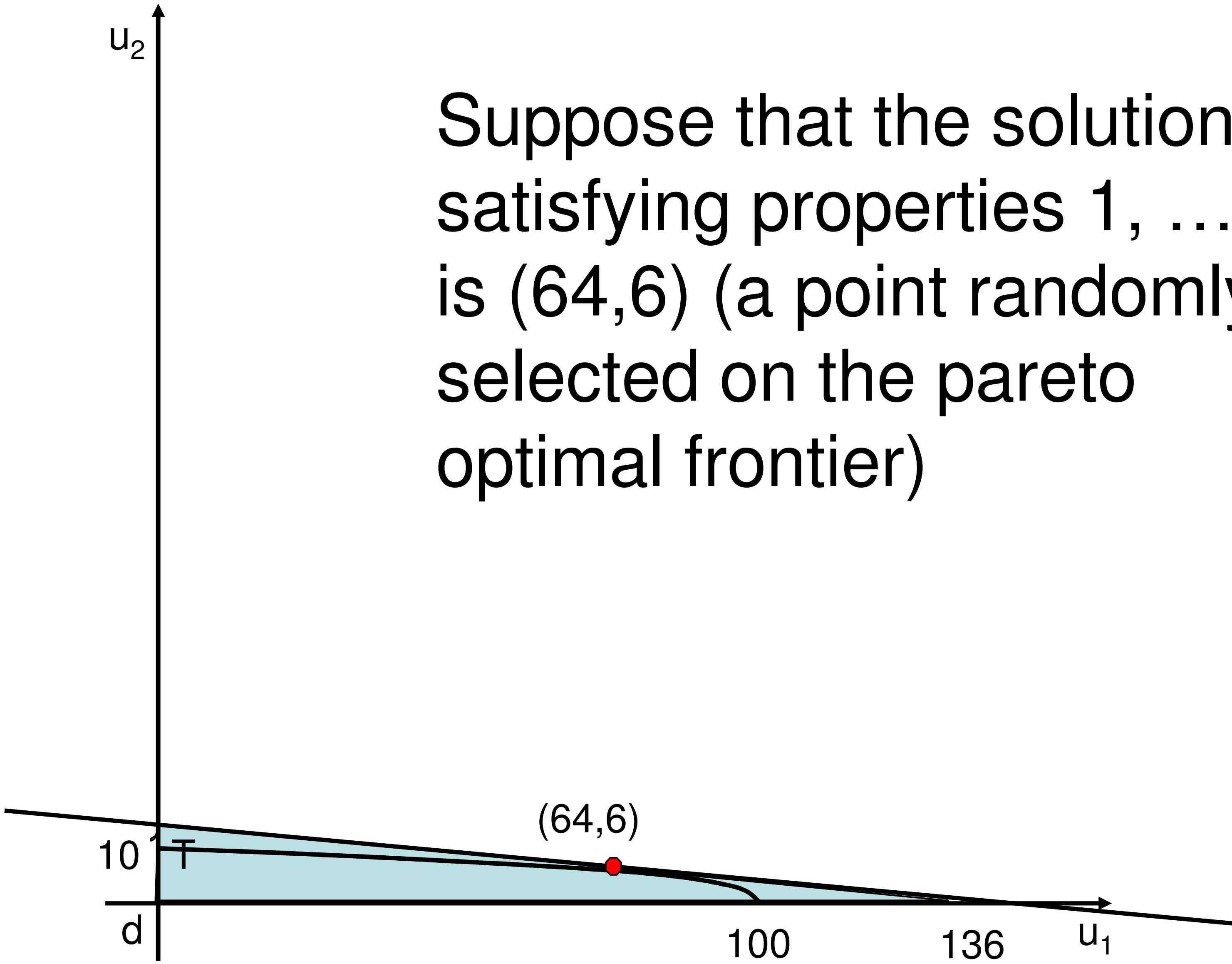
$u_1(x)=x$ and $u_2(x)=x^{1/2}$

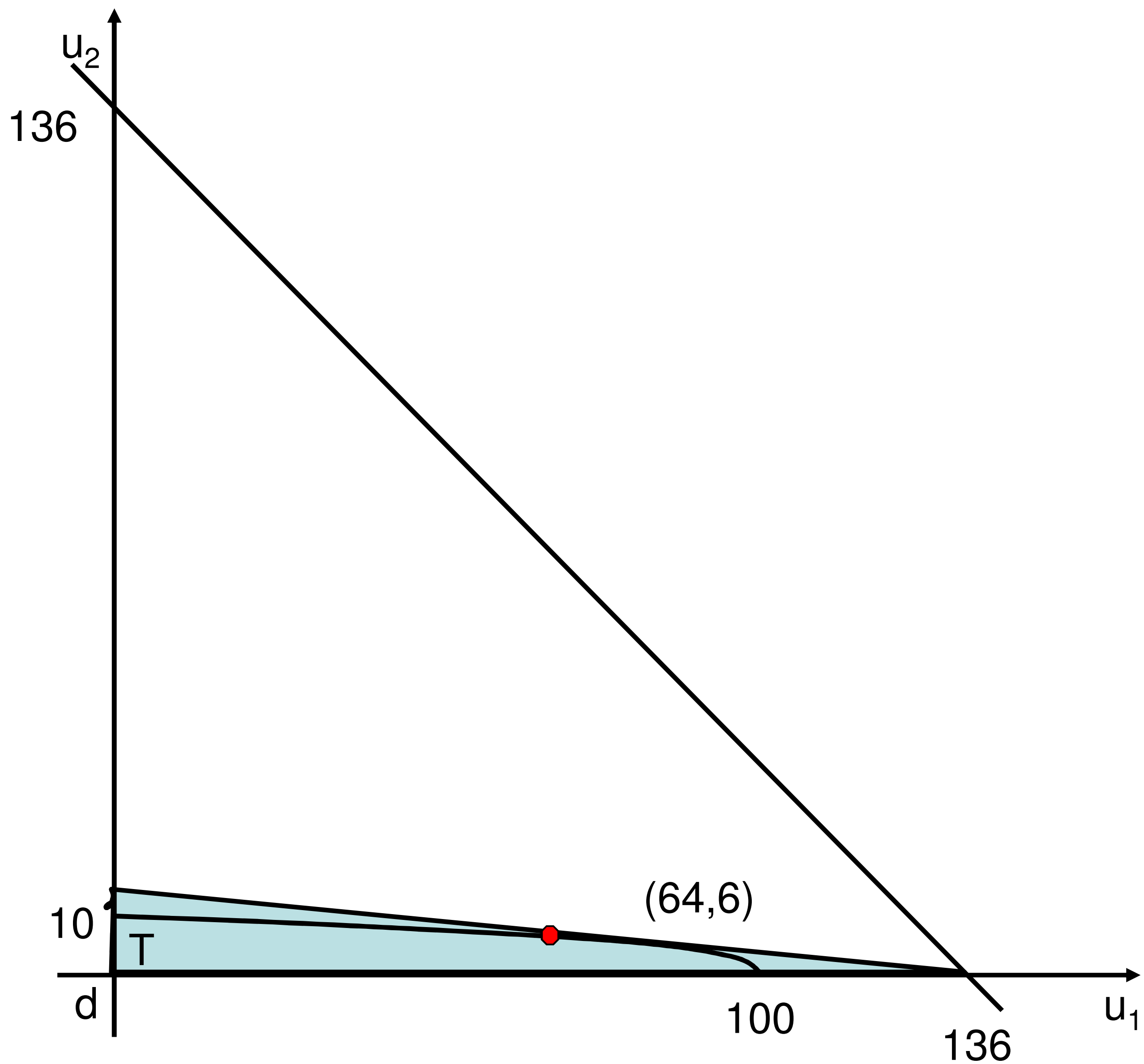




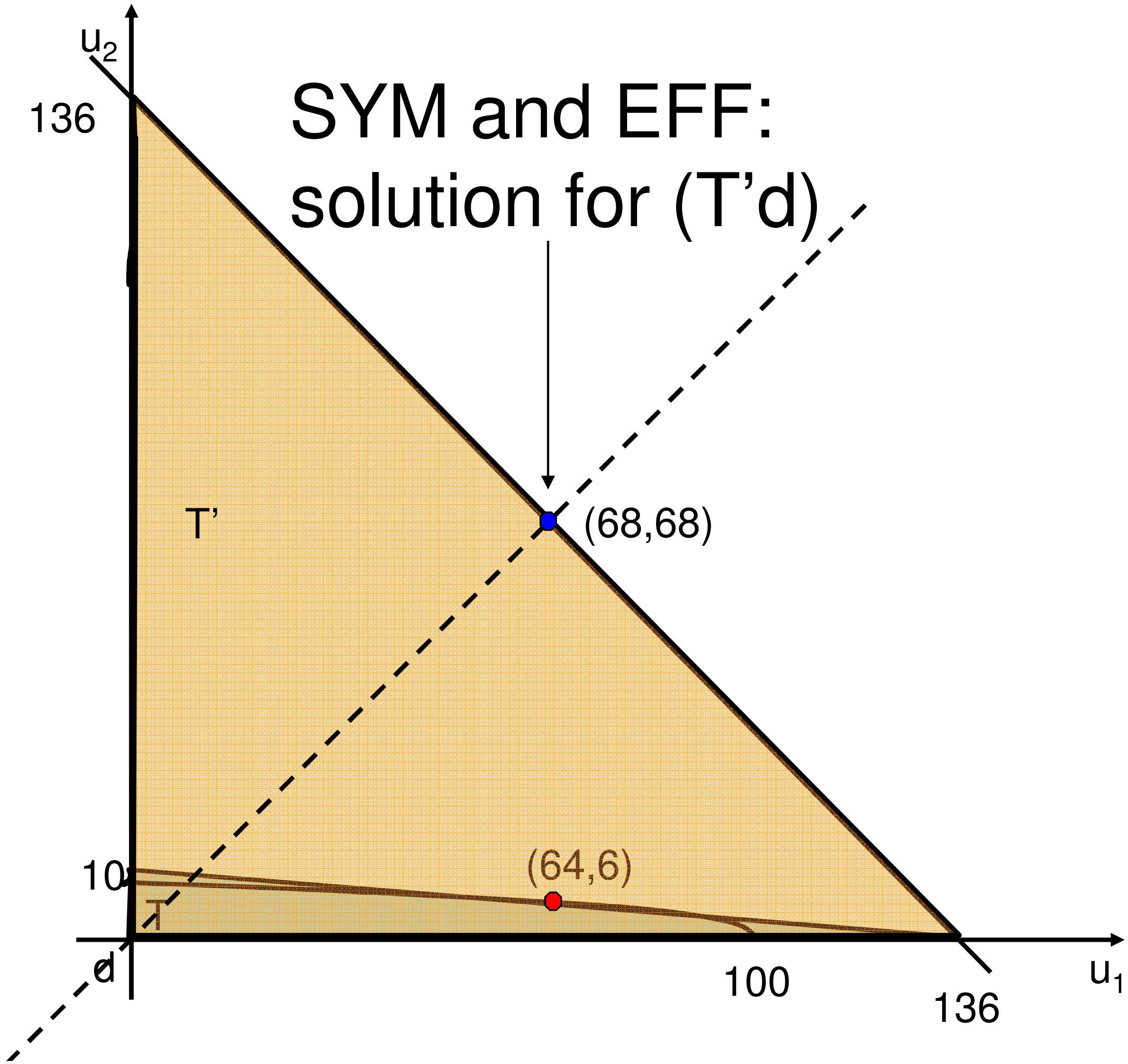
Suppose that the solution satisfying properties 1, …, 5 is (64,6) (a point randomly selected on the boundary)

Suppose that the solution satisfying properties 1, …, 5 is (64,6) (a point randomly selected on the pareto optimal frontier)

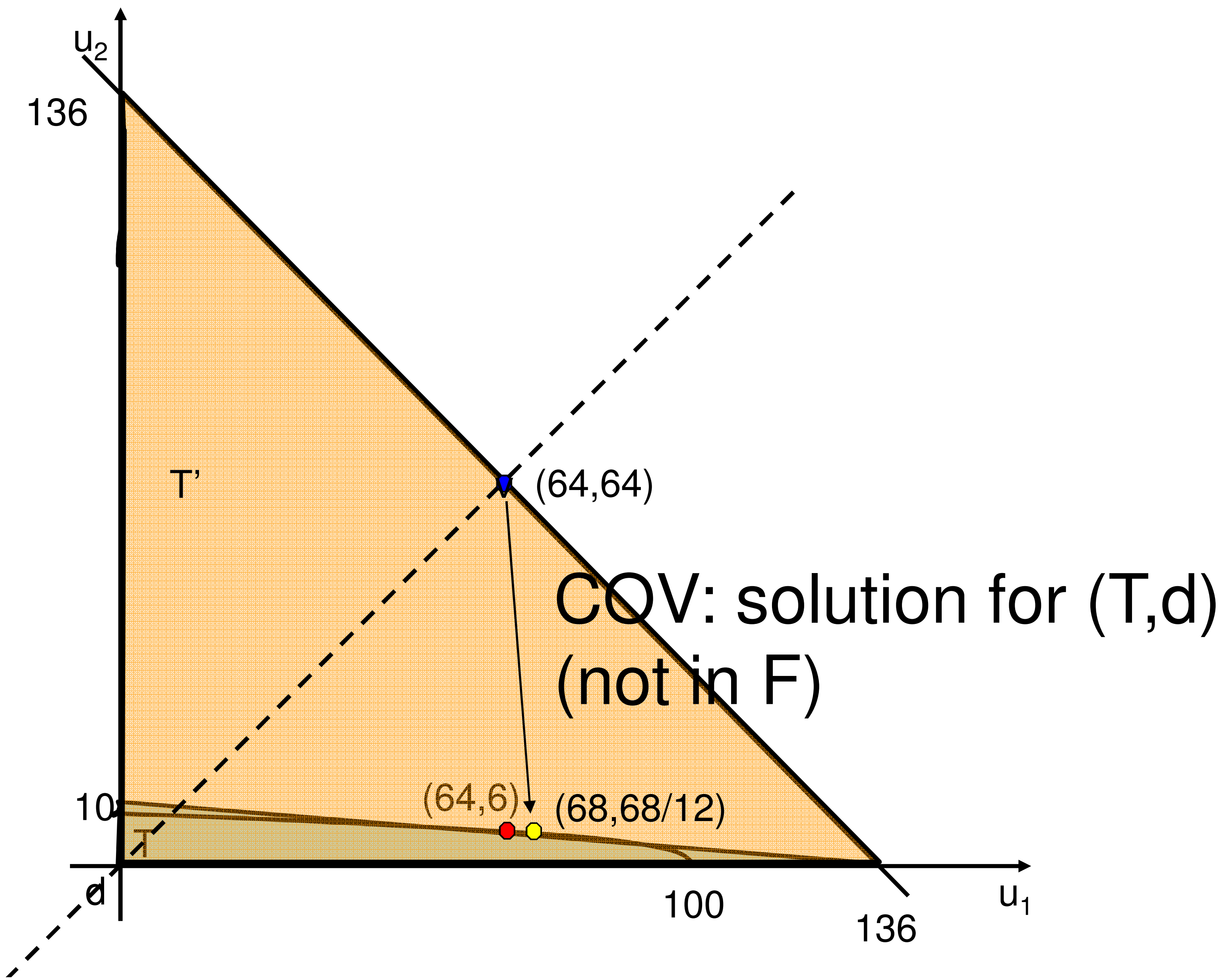




u_2
136
(64,6)
10
T
d
100
136
u_1

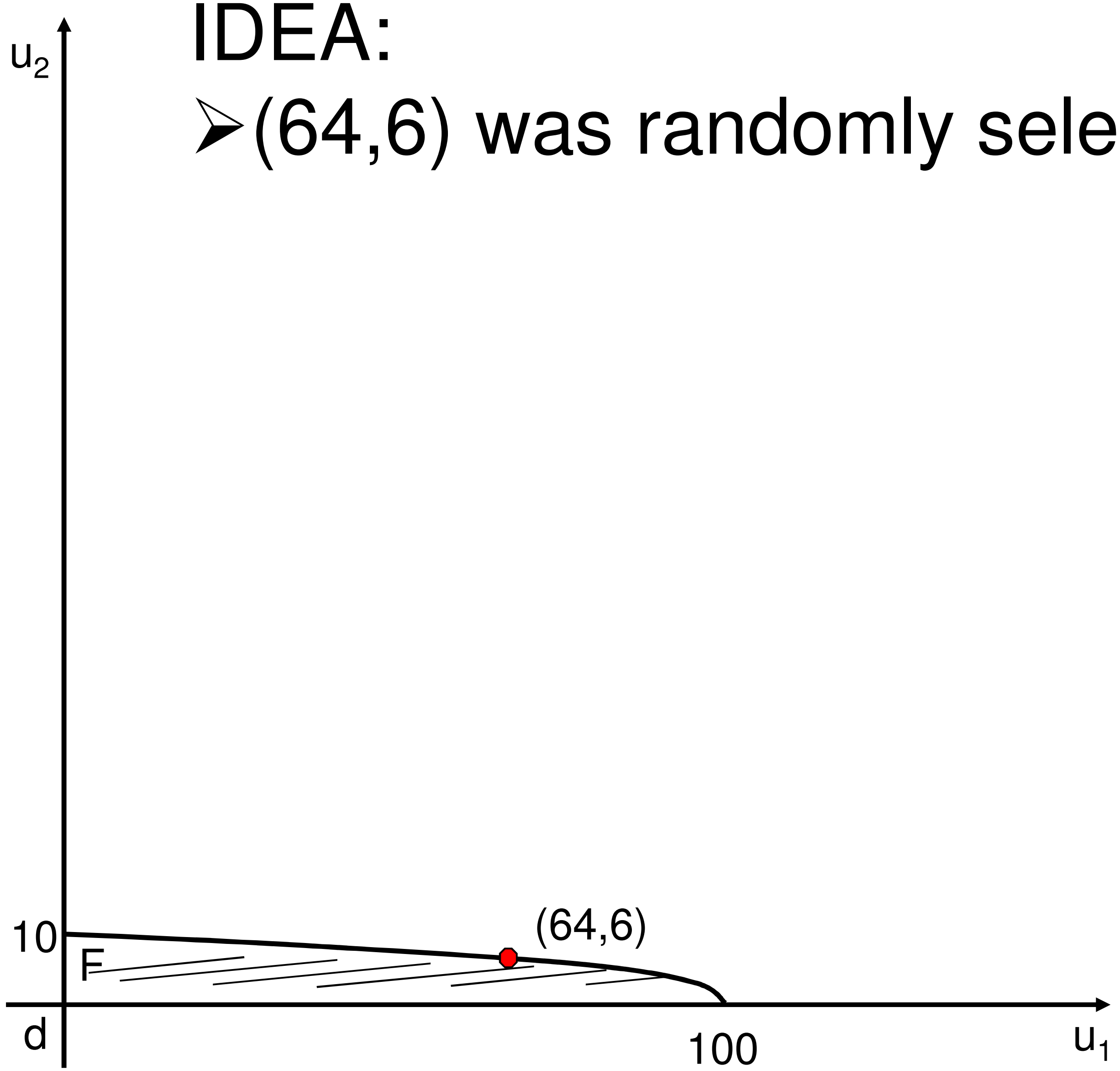


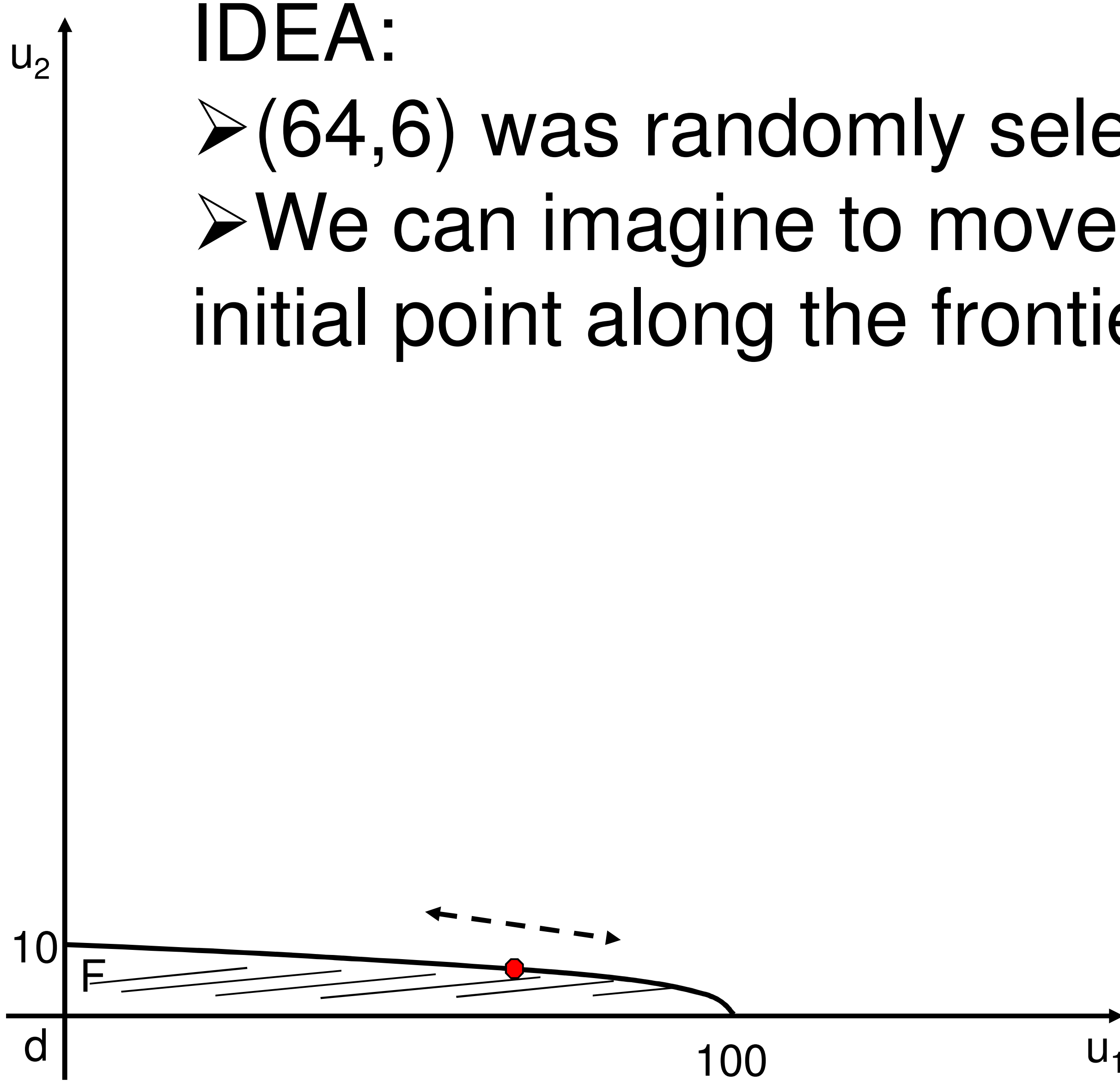
u2
136
SYM and EFF:
solution for (T'd)
T'
(68,68)
(64,6)
10
T
d
100
136
u1


u2
136
T'
(64,64)
COV: solution for (T,d)
(not in F)
10
(64,6)
(68,68/12)
T
d
100
136
u1

IDEA:

- (64,6) was randomly selected



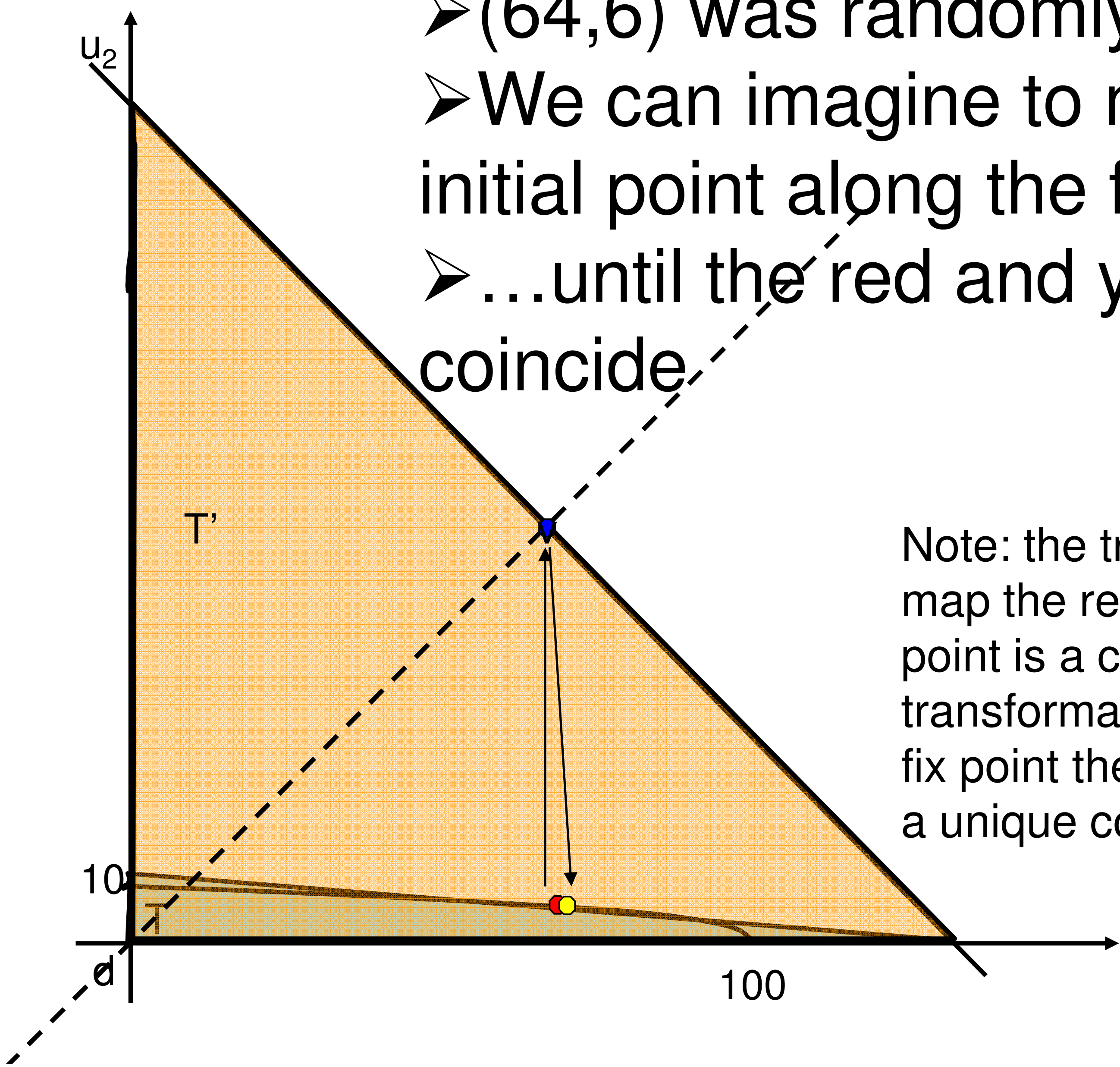


IDEA:

- (64,6) was randomly selected
- We can imagine to move the initial point along the frontier…

IDEA:

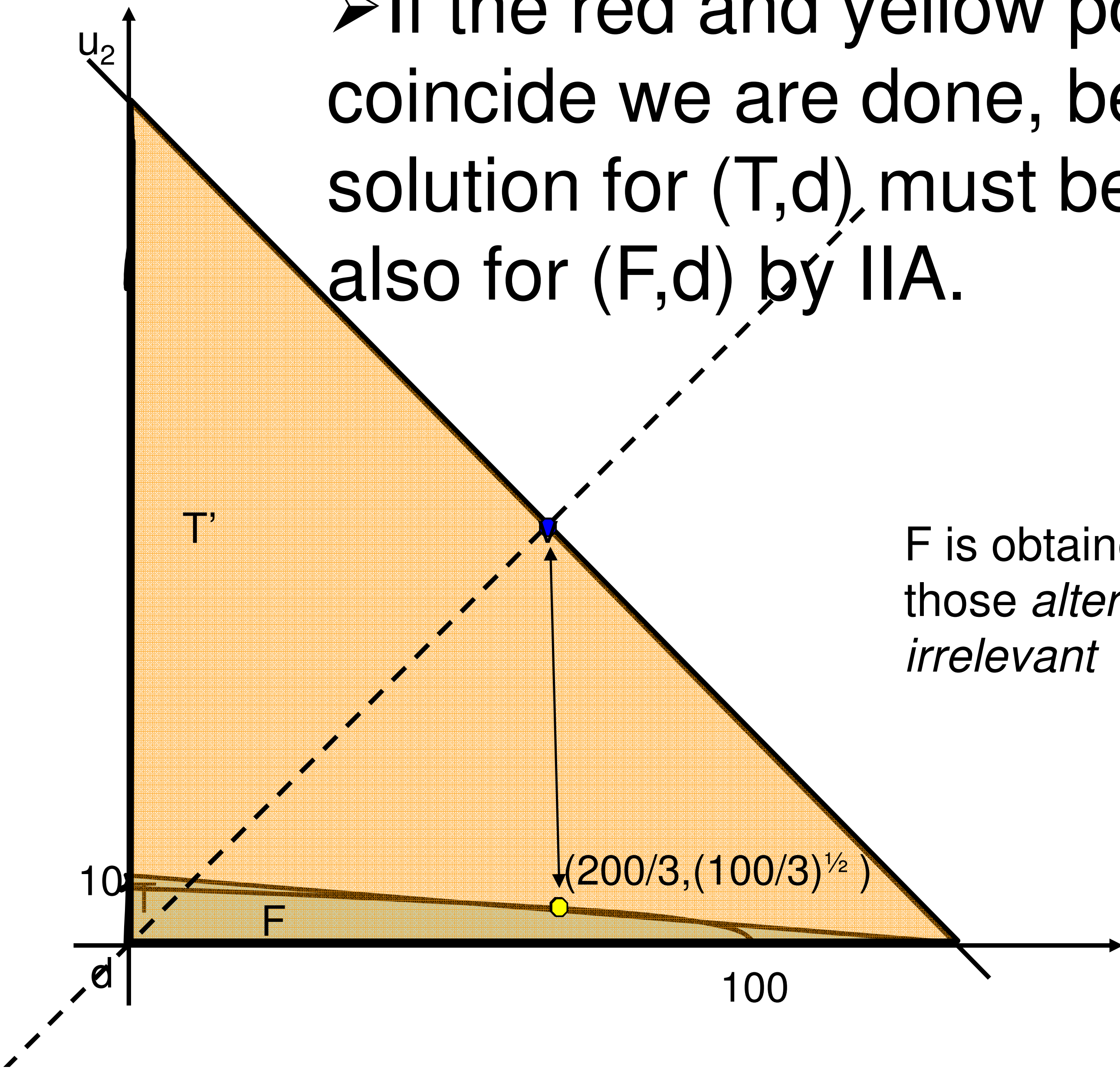
- (64,6) was randomly selected
- We can imagine to move the initial point along the frontier…
- …until the red and yellow points coincide



Note: the transformation which map the red point to the yellow point is a continuous transformation→ appropriate fix point theorem to guarantee a unique coincidence exists

IDEA:

➢If the red and yellow points coincide we are done, because the solution for (T,d) must be solution also for (F,d) by IIA.



F is obtained by T eliminating those *alternatives* that are *irrelevant*

Note:

- the hypothesis (p.1, p.2, p.3) guarantee that $F \cap (d + IR^2_{\geq})$ is non-empty, closed and bounded (it does not matter whether (F,d) is essential or not);
- $(u_1-d_1)(u_2-d_2)$ is a continuous function
- Then the Weierstrass theorem guarantees the optimization problem in (*) has <u>at least</u> one solution. As a consequence the "argmax" is non-empty.
- If the problem (F,d) is also essential, the convexity of F (and therefore of

$F \cap (d + IR^2_{\geq})$) and the strict quasi-concavity of function $(u_1-d_1)(u_2-d_2)$ on $F \cap (d + IR^2_{\geq})$ guarantee that the point of maximum is unique.

Note bis:

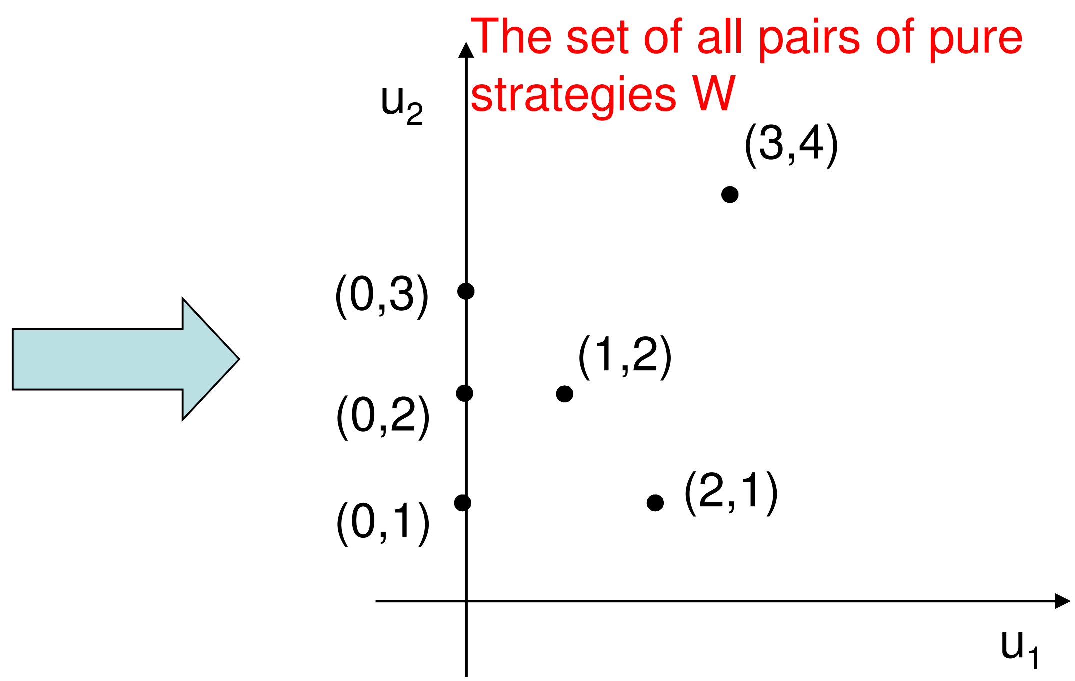
A function $f: C \to IR$, where $C \subseteq IR^k$ is convex, is said to be quasi-concave if:

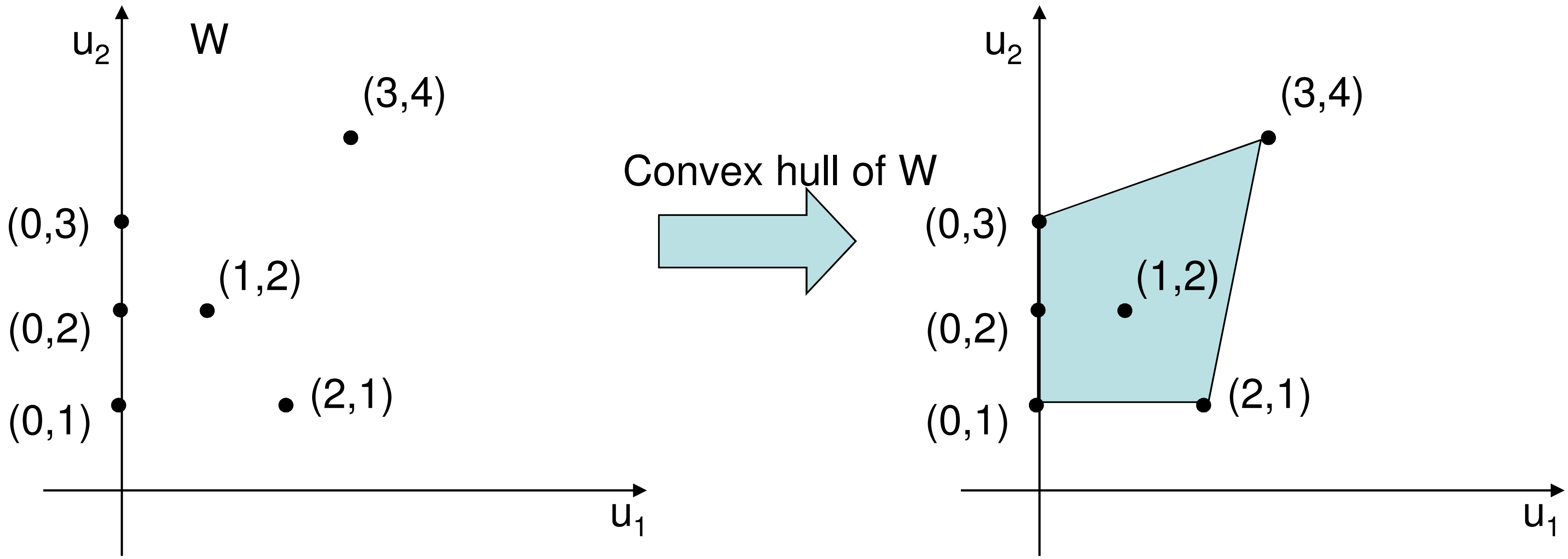
For each $c_1, c_2 \in C$, with $c_1 \neq c_2$, for each $\lambda \in]0,1[$, we have

$$f(\lambda c_1 + (1-\lambda)c_2) > \min\{f(c_1), f(c_2)\}$$

- Q: Can we describe a game in strategic form as a bargaining problem?
- suppose to have a strategic game with two players (X,Y,u_1,u_2)
  - Players can make agreements on probability distributions on pairs of pure strategies, i.e. on $X \times Y$

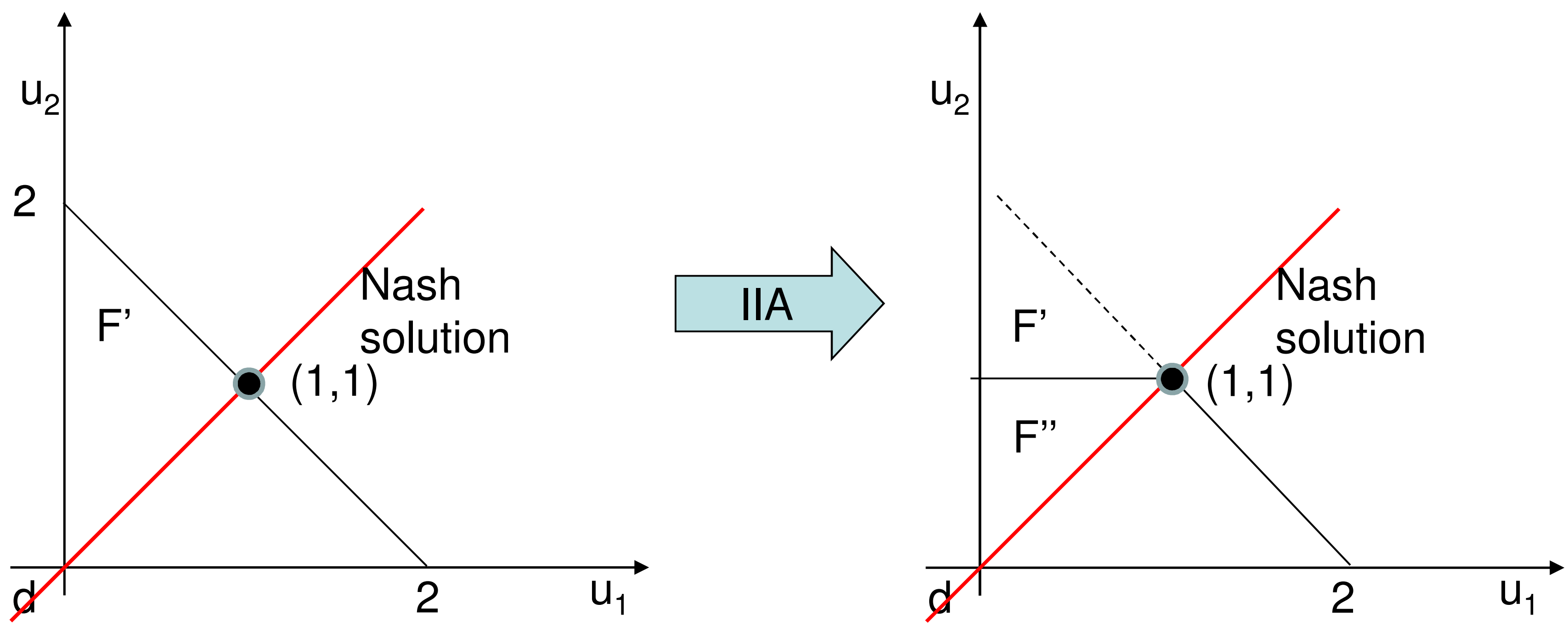
1 \ 2	**L**	**R**
T	(1,2)	(0,1)
M	(0,3)	(2,1)
B	(0,2)	(3,4)

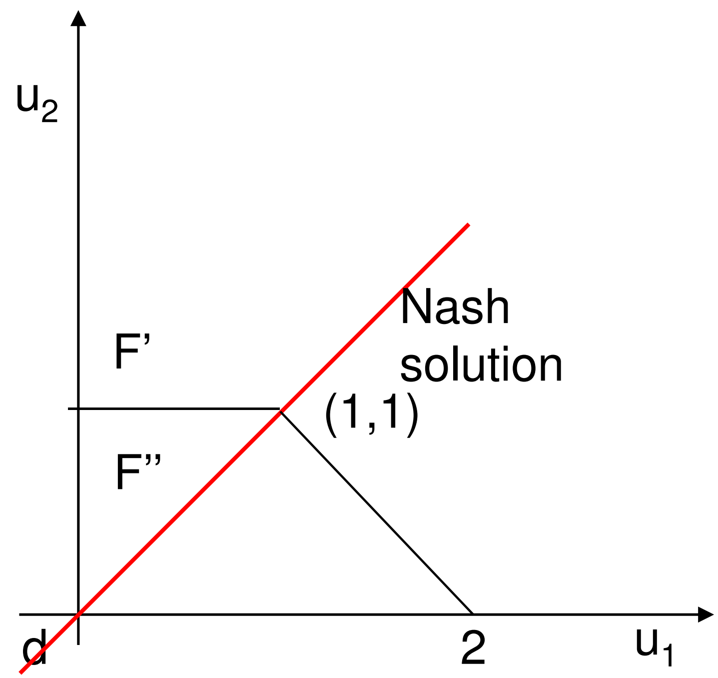




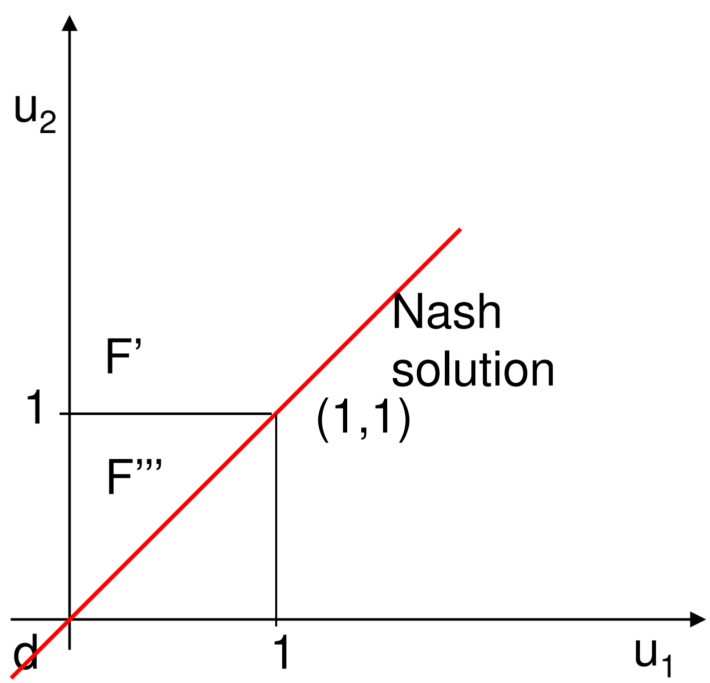
- Each point in the convex hull of W corresponds to a different correlated strategy. But…
- **Q**: which is the disagreement point?
  - Nash equilibrium? Correlated equilibrium?
  - What about uniqueness?
  - MAX-MIN? difficult to believe…
- It is not possible to go from a game in strategic form to a bargaining game in a canonical way.

- **Q**: Are there better solutions than the one proposed by Nash?
- An answer again from the axiomatic approach
- In 1975 Kalai and Smorodinsky sobstitute the axiom IIA with the axiom of Individual Monotonicity (or Ristrected Monotonicity)

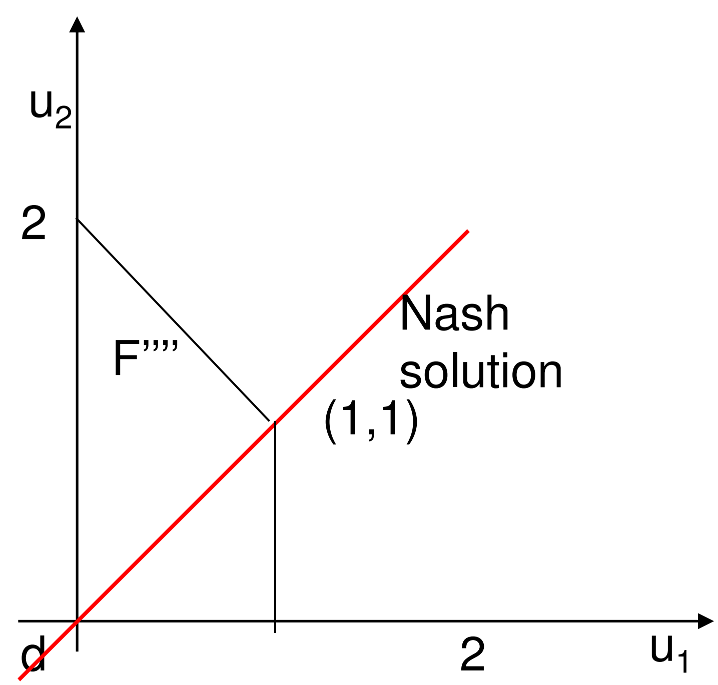




u_2
F'
F''
Nash
solution
(1,1)
d
2
u_1



u_2
F'
1
F'''
Nash
solution
(1,1)
d
1
u_1



u_2
2
F''''
Nash
solution
(1,1)
d
2
u_1

Kalai and Smorodinsky

- In order to introduce the new property of Individual Monotonicity, we first need to introduce the notion of *utopia point.* We define:
  - the point b_1 as the maximum value of u_1 on $F \cap (d + IR^2_{\geq})$
  - similarly b_2 as the maximum value of u_2 on $F \cap (d + IR^2_{\geq})$
- The point $b=(b_1,b_2)$ is said *utopia point* for (F,d) (usually it does not belong to F).

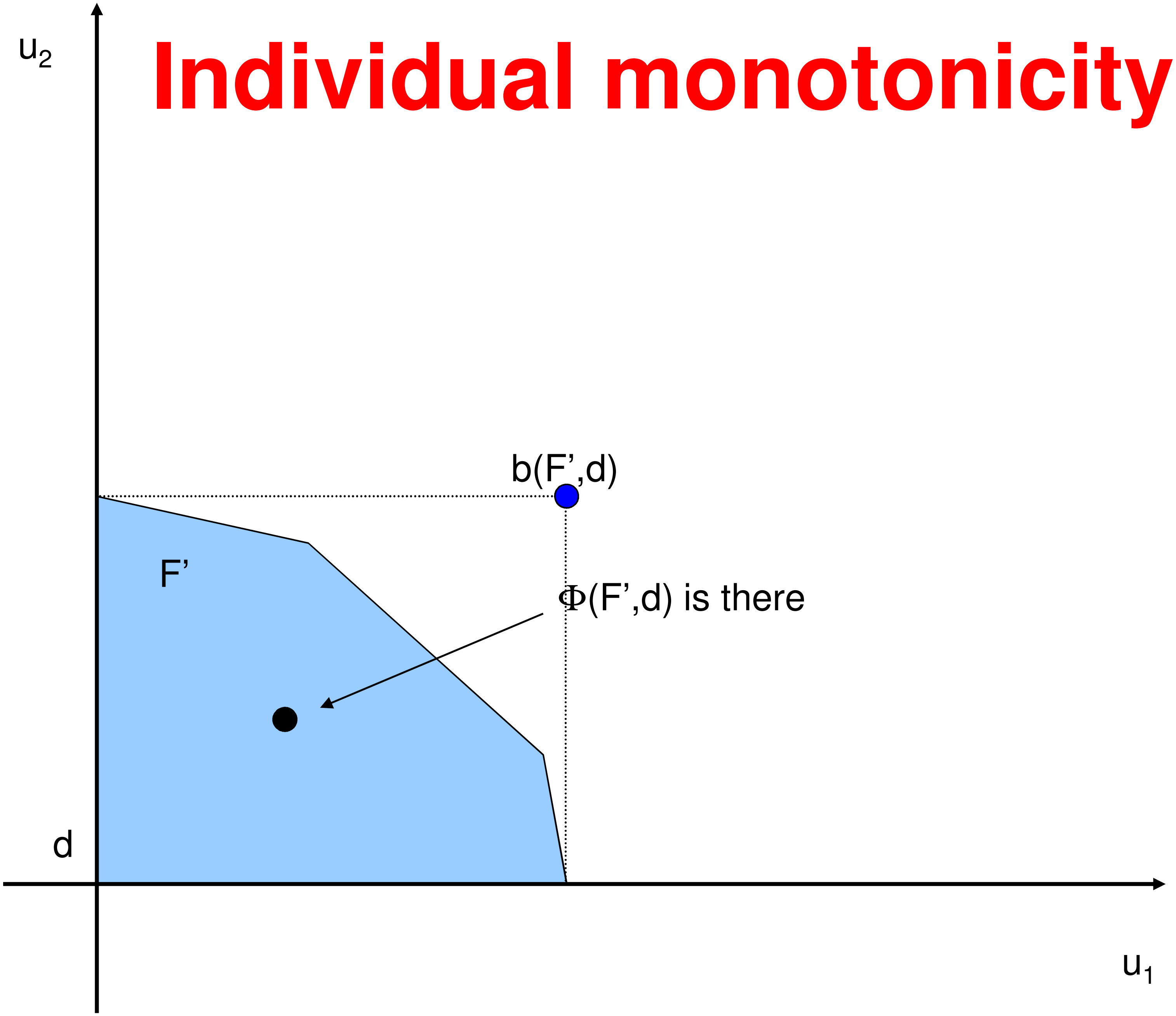
Prop. 6 Individual (or Restricted) Monotonicity (INDM)

Let $(F,d),(F',d) \in \mathbf{B}$ be such that $F' \subseteq F$.

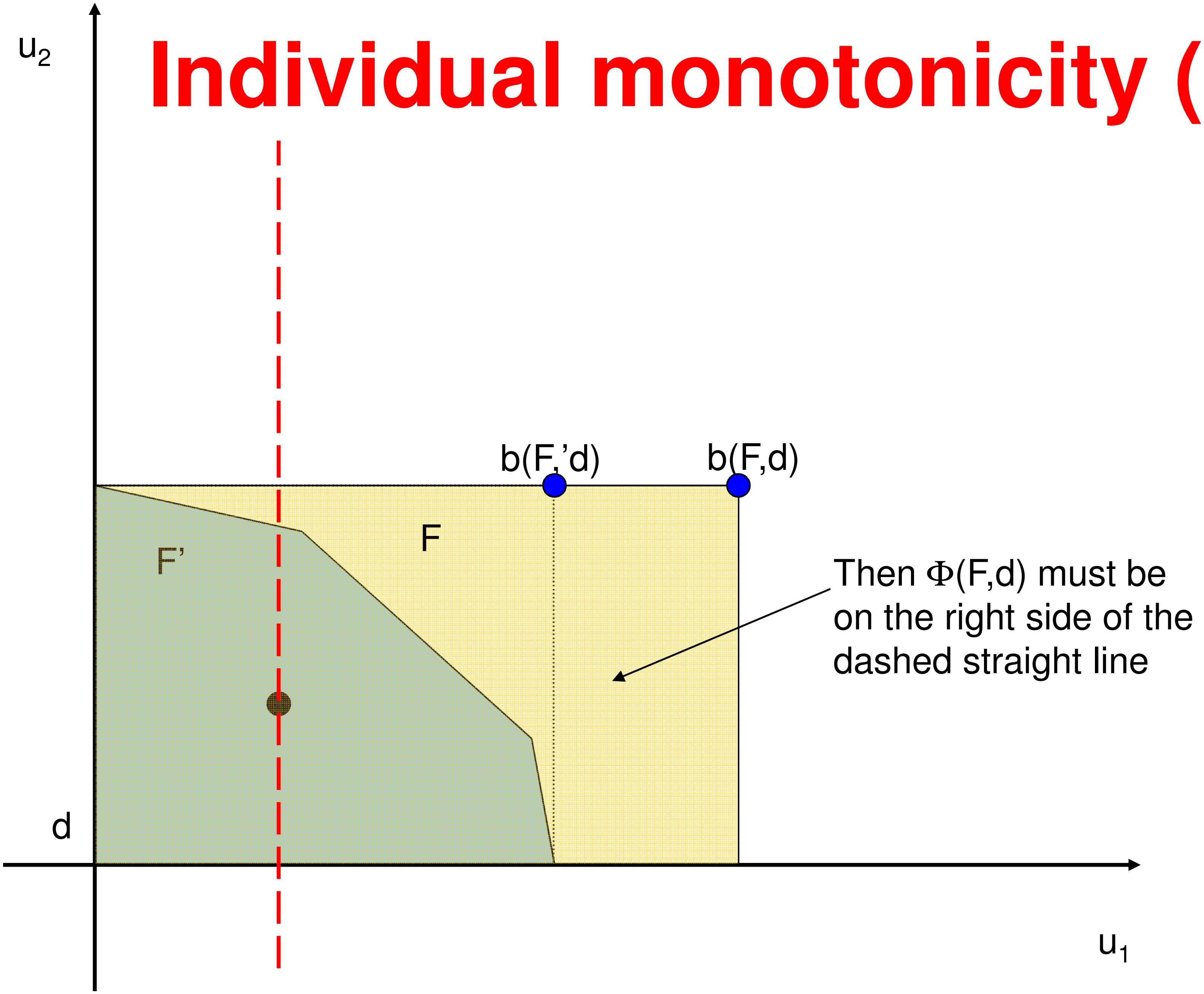
If $b_1(F,d)=b_1(F',d)$, then $\Phi_2(F',d) \leq \Phi_2(F,d)$.

If $b_2(F,d)=b_2(F',d)$, then $\Phi_1(F',d) \leq \Phi_1(F,d)$.

Individual monotonicity (1)



Individual monotonicity (2)

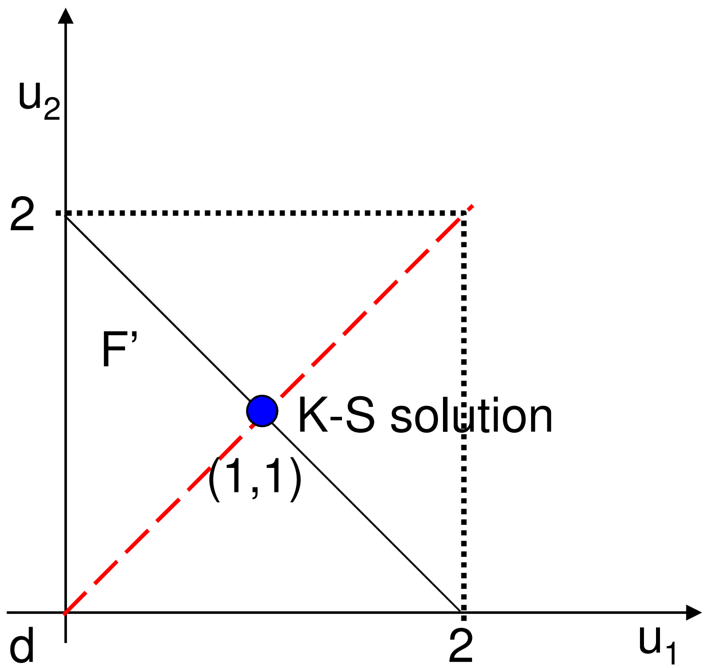


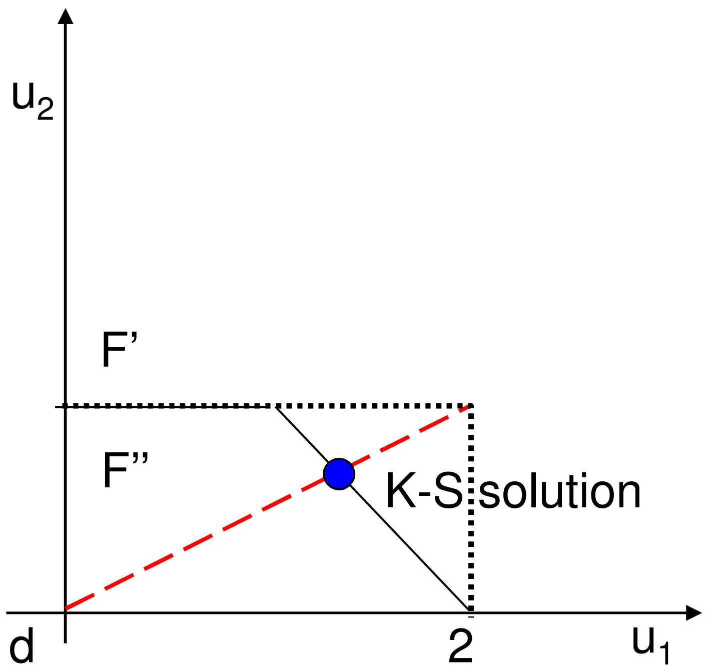
Kalai and Smorodinsky solution (1975)

Theorem

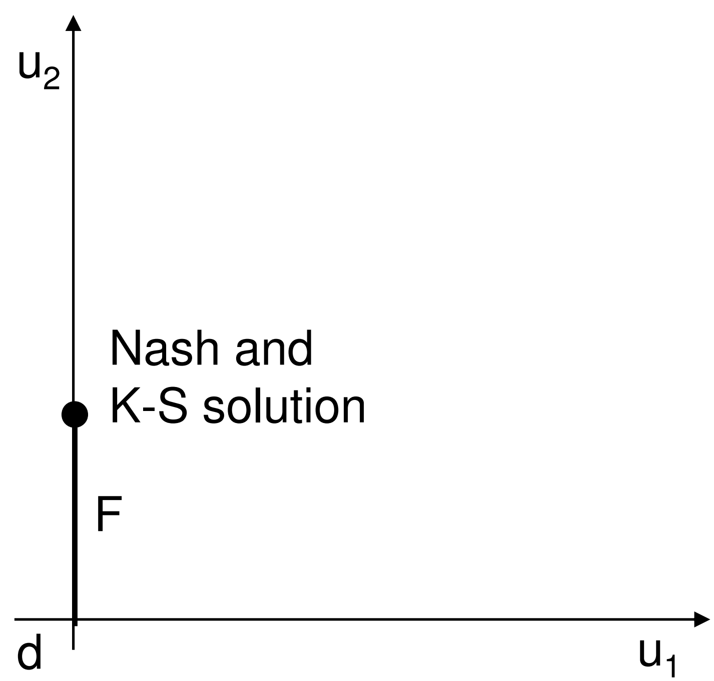
There exists one and only one solution Φ defined on **B** which satisfies properties EFF, INDR, SYM, COV and INDM. Moreover, if $(F,d) \in$ **B** is essential, we have that such a solution is given by the unique Pareto optimal point on the segment which connects the disagreement point d with the utopia point b(F,d).

Kalai and Smorodinsky solution on two examples





- For both solutions (Nash (1950) and Kalai and Smorodinsky (1975)) the solution of a non-essential problem is determined by the EFF property.
- Note that in a non-essential bargaining problem, the set of points of F which satisfy the INDR property, i.e. $F \cap (d + IR^2_{\geq})$, trivially are a vertical or horizontal segments (possibly collapsed to a single a point!).



Non Transferable Utility (NTU)-games

- An NTU-game is a pair (N,v) where
  - $N=\{1,2, \ldots, n\}$
  - V is a map assigning to each $S \in 2^N \setminus \{\varnothing\}$ a subset V(S) of $\mathbb{R}^S$ such that the following properties hold:
- (p.1) V(S) is a non-empty closed subset of $\mathbb{R}^S$.
- (p.2) V(S) is *comprehensive*, i.e. if $u \in V(S)$ and $v \in \mathbb{R}^S$ such that $v \leq u$, then $v \in V(S)$
- (p.3) $\{u \in \mathbb{R}^N \mid u_i \geq v(i),\ u \in V(N)\}$ is bounded, where $V(\{i\})=(-\infty, v(i)]$

Interpretation

- The elements of N are players who can cooperate
- If coalition S forms, then each of the payoff vectors $u \in V(S)$ is attainable, giving reward (utility) u_i to player $i \in S$

Example

A 2-person bargaining game (F,d) (with F comprehensive) can be seen as a 2-person NTU-game ({1,2},V) where

$V(\{1\})=(-\infty, d_1]$

$V(\{2\})=(-\infty, d_2]$ $V(\{1,2\})=F$

Example

Three voters 1,2,3 have to decide between two alternatives a_1,a_2. The majority decide. The utilities of the voters:

	a_1	a_2
1	5	1
2	2	3
3	4	3

We may consider a 3-person NTU-game (N,V) where

N={1,2,3}

V(1)=(- ∞, 1] V(1,2)=compr_hull{(5,2),(1,3)}

V(2)=(- ∞, 2] V(1,3)=compr_hull{(5,4),(1,3)}

V(3)=(- ∞, 3] V(1,2)=compr_hull{(2,4),(3,3)}

V(1,2,3)=compr_hull{(5,2,4),(1,3,3)}

A non-cooperative approach to bargaining: the *ultimatum game*

- Two players bargain to divide a fixed amount between them (a “pie”).
- Player I (proposer) offers a division of the “pie”
- Player II (responder) decides whether to accept it
- If accepted, both player get their agreed upon shares
- If rejected players receive nothing.

What do game theorists say?

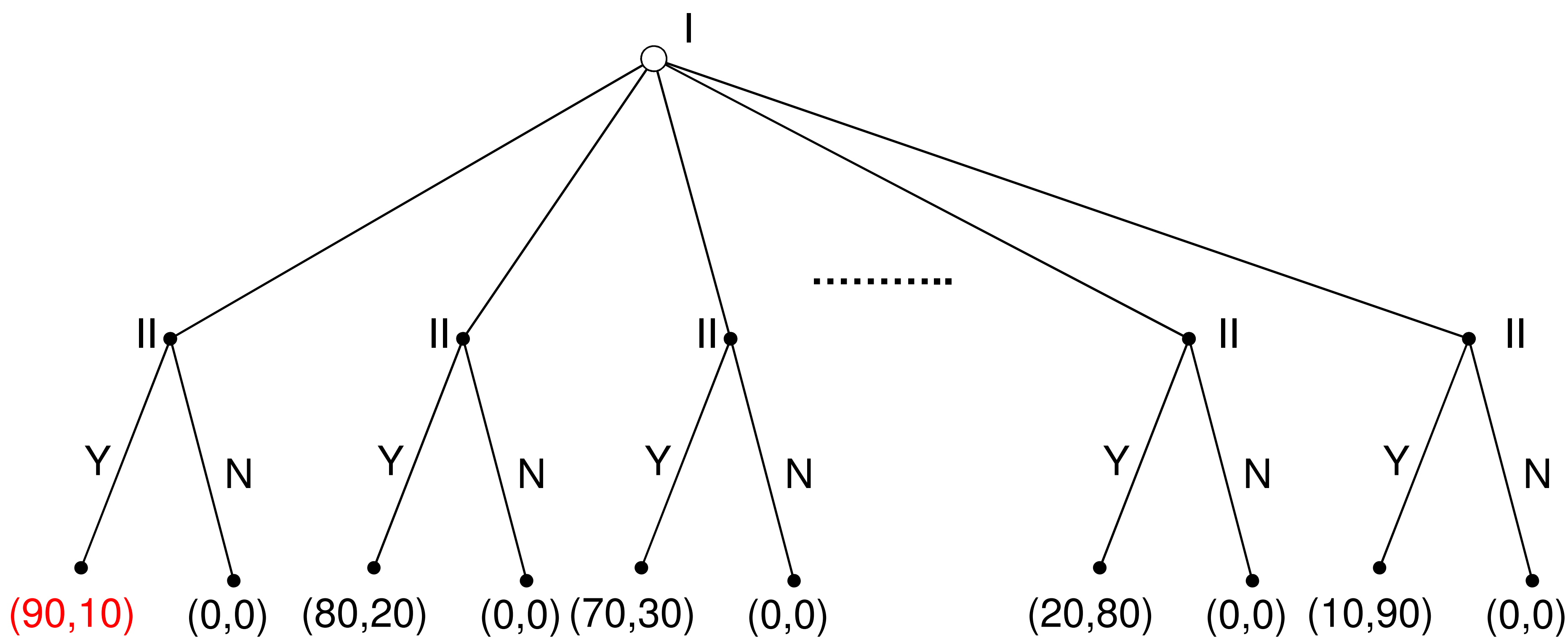
- Ariel Rubenstein (1982)
- showed that there exist a unique subgame perfect Nash equilibrium to this problem giving (π - ε , ε)
- So the rational solution was predicting that proposer should offer the smallest possible share and responder would accept it.

➢100 euros to divide (in 10 pieces of 10 euros each …)

➢Player I may propose between 1 and 9 pieces

➢Utilities: if (x,y) is an allocation (x euros for player I and y euros for player II), we assume

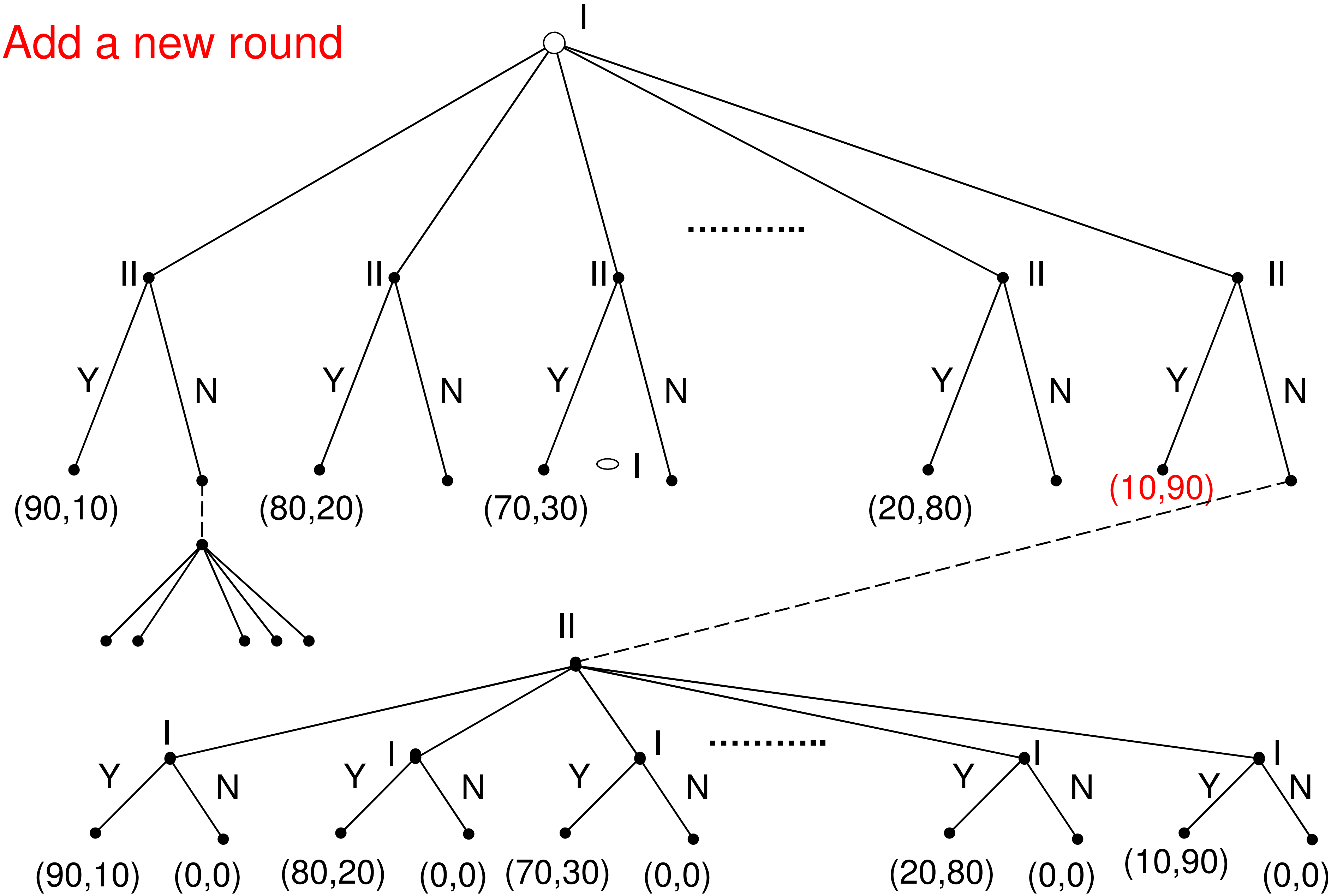
$$u_I(x,y)=x \text{ and } u_{II}(x,y)=y$$



Experimental data is inconsistent !

- Güth, Schmittberger, Schwarze (1983)
  - They did the first experimental study on this game.
  - The mean offer was 37% of the “pie”
- Since then several other studies has been conducted to examine this gap between experiment and theory.
- Almost all show that humans disregard the rational solution in favor of some notion of “fairness”.
  - The average offers are in the region of 40-50% of the “pie”
  - About half of the responders reject offers below 30%

Add a new round



Infinite number of offers and counter-offers

- Imagine a new game in extensive form where a third round is added where player I has again the advantage of the last decision.
- Then a fourth round, and then a fifth one and so on…
- Introduce a discount factor (i.e. a number which a future cash flow, to be received at time T, must be multiplied by in order to obtain the current present value)
- Suppose that the discount factor is the same for both players
- Suppose that players are risk-neutral

→ We obtain a division close to (50%,50%) (still asymmetry due to the fact that player I moves first)

Mechanism design

- setting up the rules of the games, such as voting procedures or auctions rules, in order to induce a certain outcome, given that players act rationally.
- For example, game theory can help to understand what type of penalties, rewards or tax system are most effective to induce industrial companies to apply environmentally friendly production methods.
- In the context of bargaining, common goals of mechanism design are **maximising social welfare** (i.e., the sum of utilities of the players) or maximising revenues.

Example

- The Vickrey (1961) auction model was later expanded by Clarke (1971) and Groves (1973) to treat a public choice problem in which a public project's cost is borne by all agents, e.g. whether to build a municipal bridge.
- The resulting "Vickrey–Clarke–Groves" mechanism can motivate agents to choose the socially efficient allocation of a public good even if agents have privately known valuations.

Bargaining in AI

- *Software agent* is an autonomous software program which operates on behalf of its owner. Software agents have a certain goal, e.g. is to maximise a given utility function. They can usually learn from experience and adapt their behaviour given feedback from the environment, without any human intervention.
- When multiple software agents interact, the entire system is called a multi-agent system.
- Simplifying assumptions frequently made in game-theoretical analyses, such as assumptions of *rationality* and *common knowledge*, do not need to be made if the behaviour negotiating agents is modelled directly, for instance using techniques from the field of artificial intelligence (AI).

Bargaining in AI

- AI techniques commonly used to develop a negotiation environment consisting of intelligent agents are: *evolutionary algorithms*, *reinforcement learning* and *Bayesian beliefs*.
- Using these techniques, agents are able to learn from experience and adapt to changing environments.
- This learning aspect is essential for automated negotiation settings (where software agents, bargain on behalf of their owners), especially when the behaviour of competitors and the payoffs are not known in advance.

Example

J.R. Oliver. A machine learning approach to automated negotiation and prospects for electronic commerce. Journal of Management Information Systems, 13(3):83–112, 1996.

- Oliver (1996) was the first to demonstrate that a system of adaptive agents can learn effective negotiation strategies using *evolutionary algorithms*.
- Binary coded strings represent the agents' strategies.
- Two parameters are encoded for each negotiation round: a threshold which determines whether an offer should be accepted or not and a counter offer in case the opponent's offer is rejected (and the deadline has not yet been reached).
- These elementary strategies were then updated in successive generations by a *genetic algorithm*.

Examples

[1]D.D.B. van Bragt, E.H. Gerding, and J.A. La Poutr´e. Equilibrium selection in alternating-offers bargaining models: The evolutionary computing approach. The Electronic Journal of Evolutionary Modeling and Economic Dynamics (e-JEMED), 1, 2002.
[2] G. Dworman, S.O. Kimbrough, and J.D. Laing. Bargaining by artificial agents in two coalition games: A study in genetic programming for electronic commerce. In Genetic Programming 1996: Proceedings of the First Annual Conference, pages 54–63. The MIT Press, 1996.

- In [1], a related model of adaptive agents was investigated. Here, a systematic comparison between game theoretic and evolutionary bargaining models was made.
- Dworman et. al [2] studied negotiations between three players. If two players decide to form a coalition, a surplus is created which needs to be divided among them. The third party gets nothing. Of course, all three players want to be part of the coalition in this case. Moreover, they also want to receive the largest share of the bargaining surplus.
- Genetic programming was used in this paper to adapt the offers and to decide whether to form a coalition or not. A comparison with game theoretic predictions and human experiments was made.

Argumentation-based negotiation

- An alternative approach to automated negotiation is the use of dialogues or argumentation to resolve conflicts. In recent years, this field has received increasing interest within the agent community.
- The idea is that the agents are able to provide "meta-information" on why they have a particular objection to a proposal.
- information is exchanged, but without fully disclosing each others' preferences.

- Argumentation can also be used to influence the “preferences” (but how preferences are modelled?) and beliefs of other players.
- a player’s preferences can be influenced upon receipt of new information.
- The negotiation process then not only consists of dividing the surplus, but also of gathering information.

Example

I. Rahwan, L. Sonenberg, and F. Dignum. Towards interest-based negotiation. In Proceedings of the Second International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS), Melbourne, Australia, pages 773–780. ACM Press, 2003.

- One player may influence another player's preferences by discussing the underlying motivations and interests behind adopting certain (sub-) goals.
- For example, a buyer may want to negotiate a flight ticket with a travel agent for the more fundamental goal of travelling to Paris.
- If the fundamental goal is known to the travel agent, she can suggest a train ticket as an alternative means to satisfy the same goal.