Exercise 1 *You are playing the following game. It is given an indivisible object. Players are two. Each one has to make a guess about the price of that good. One player has to give his guess, then the second player (you), after having listened what the other player has said, must give his guess. The player which will get closer to the real price will win some amount of money (if there is a tie, the sum is divided evenly).*

Which rule of behavior (strategy?) would you use? How would you model this situation? Do you think that the situation is well specified, or you would need further (or less...) details to build a sensible model for it?

Exercise 2 *Prove that in the beauty contest game the strategy profile* $(1, \ldots, 1)$ *is a Nash equilibrium.*

Prove that the strategy 67 dominates strategy 68, specifying the kind of dominance that you have, and the assumptions that you need to prove this result.

Exercise 3 *Provide two examples of games s.t. in the first it is better for player I that player II is rational and intelligent, and in the second it would be better for player I that player II chooses his strategy in a random way.*

Please specify also what is your interpretation of "chooses his strategy in a random way".

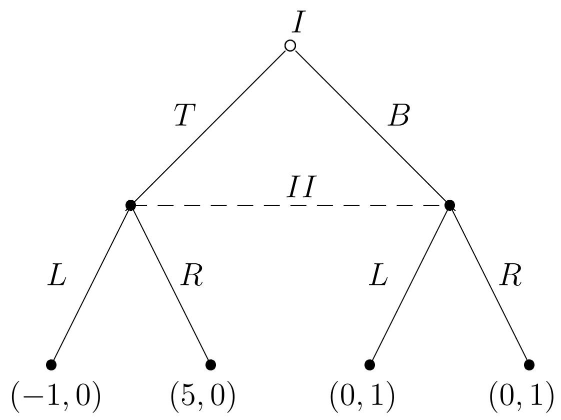
Exercise 4 *Let G be a finite game with two players. Let $(\bar{x}, \bar{y})$ be a Nash equilibrium for G. Prove that $(\bar{x}, \bar{y})$ is also an equilibrium for the mixed extension of G.*

Specify in a detailed way what is the correct formalization of the sentence above.

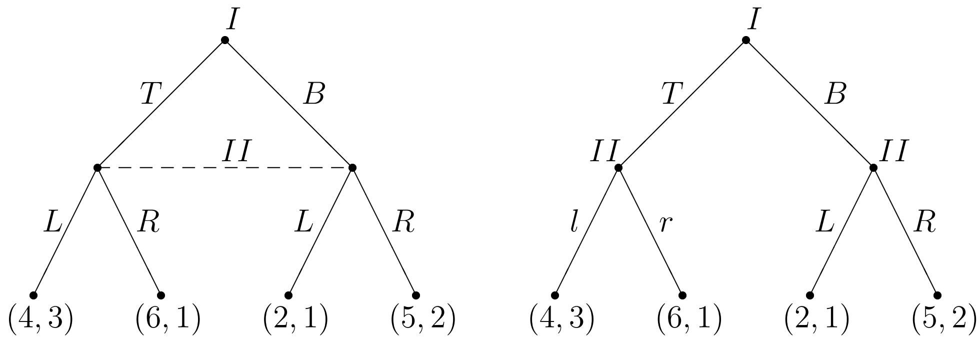
Exercise 5 *Let $G = (X, Y, E, h, u, v)$, and let $\tilde{G} = (X, Y, E, h, \tilde{u}, \tilde{v})$, where $\tilde{u} = \phi \circ u$ and $\tilde{v} = \psi \circ v$, with $\phi, \psi : \mathbb{R} \to \mathbb{R}$ are strictly increasing.*

Prove that $(\bar{x}, \bar{y})$ is a Nash equilibrium for G iff it is so for $\tilde{G}$.

Exercise 6 *Find all SPE for the following game. Which strategy would you choose, being player I?*



Exercise 7 *Consider the two following games:*



Write the strategic form of these games and find their Nash equilibria (in pure strategies). Comment.

Exercise 8 *Consider the sealed-bid second price auction (or Vickrey auction). Write a sensible game form for it. Assume that the preferences of player i (who are the players?) are characterized by his valuation v_i. Prove that bidding v_i is a dominant (in which sense?) strategy for player i.*

Exercise 9 *Discuss what is said at page 4 column 1 and at page 6, first paragraph of the conclusions, in the paper "Game models for cognitive radio algorithm analysis" by J. Neel, J. Reed, and R. Gilles.*

In your opinion the authors mean that in some instances one should discard the efficiency requirement? Are you able to give reasonable examples in which a planner could not be so much interested in the efficiency of the result? How would you cast their comments in the "mechanism design" context? Can you model what is described in the paper using the formalism of "mechanism design"?