Introduction to Game Theory and Applications, IV

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Luxembourg, June 2010

Summary

Dynamics?

Well, Nash equilibrium.

Idea of balance of forces (uhm... utility and disutility?)

Rest point of a dynamic process: no reason to move from it.

Seems quite close to the idea of Nash equilibrium.
Let's dig.

Best reply dynamics

After all, the existence thm was based on the idea of a fixed point for a (multivalued) map.

An implicit dynamics. Let's write it explicitly

$$\begin{cases} (x_1, y_1) & = & (\hat{x}, \hat{y}) \\ (x_{n+1}, y_{n+1}) & \in & R_I(y_n) \times R_{II}(x_n) \quad \forall n \in \mathbb{N} \end{cases}$$

Remarks on best reply dynamics

$R_I(y_n) \times R_{II}(x_n)$ can be empty. Not possible for a finite game, or for mixed extension of a finite game (maximizing continuous function on a nonempty compact set: Weierstrass' theorem).

But, even in this setting, *multivalued*. So, we need a selection procedure. Don't forget it.

We could consider some variants (alternation of players).

It works for the simplest Cournot model. It was the idea of dynamics, of repeated adjustment that Cournot was proposing.

Best reply dynamics for finite games

We know that, for a metric space, convergence of a sequence means that it is eventually constant.
So, we can expect the following:

- either the dynamics gets trapped into a point (that will be a NE, very good!)
- or it will be trapped into a cycle

Of course, the last thing *must* happen with matching pennies, since there is no NE.

An example

$I \backslash II$	L	C	R
T	$0, 0$	$1, 1$	$3, 0$
M	$1, 1$	$0, 0$	$0, 0$
B	$0, 3$	$0, 0$	$0, 0$

(M, L), (T, C) are NE.

(M, L) "attracts" dynamics starting at: (M, L) (obviously), (T, L) and (M, R).

(T, C) "attracts" dynamics starting at: (T, C) (obviously), (T, R) and (B, C).

(T, L) AND (M, C) are a cycle. Which "attracts" also (B, R).

Introduction to the fictitious play

Finite games need not to have NE!
So, let's move to "mixed strategies"
The idea of "fictitious play" (Brown):
One keeps track of the frequencies of play (a naïve statistics).
And plays the BR to the mixed strategy identified by those relative frequencies.

In "matching pennies" it works fine.

Not a surprise: thm by Julia Bowman (first woman mathematician to be elected to the National Academy of Sciences, in 1975) guarantees that fictitious play will always converge to a NE. But for zero-sum games ("matching pennies" is a zero-sum game).

Shapley's example

$I \backslash II$	L	C	R
T	$0, 0$	$1, 2$	$2, 1$
M	$2, 1$	$0, 0$	$1, 2$
B	$1, 2$	$2, 1$	$0, 0$

Fictitious play does not converge. It continues to oscillate (with "periods" of increasing length).

Nice connection with correlated equilibria (for interested people).

Positive (small) result for no-zero sum games: it converges for 2×2 games (Miyasawa).

Miyasawa was mistaken?

Monderer and Sela offer an example of a 2×2 game without the "fictitious play property". That is, fictitious play may not converge.

And so?

Simple: it depends on the tie-braking rule. If one asks for convergence for any tie-braking rule, there is a counterexample:

I \ *II*	L		R	
T	0	1	0	0
B	0	0	0	1

Dynamics?

Fictitious play as an algorithm to compute Nash equilibria.

No real dynamics! Players are not involved in making choices repeatedly!

Is there any difficulty, maybe?

YES, big difficulty. Our players are “tremendously” intelligent. The rule described by fictitious play is too naïve.

Stubborn players

As was naïve the best reply dynamics (and the idea of Cournot). Think of the duopolists: every year they make a decision, based on the assumption that the other duopolist will not change his strategy. But this fact is provenly false at every stage, as they can check. Stubborn players (being stubborn is far from being intelligent; similar situation as in the "centipede" game).

Even with Walras and his tâtonnement: "prix criés au hasard " by an auctioneer. But no trade! Just computations. Trades happen only "at the end", when prices have reached equilibrium.

Incomplete information

Not all of the details are common knowledge.

Learn characteristics of the other player(s) via repeated interaction.

Your "moves" as costly signals!

Build a reputation. As in the model of Kreps, Milgrom, Roberts and Weber: it is important for the intelligent type of player I to pretend to be a TFT machine.

Incomplete information can be on many things

How long it will last the interaction?

Example: after every stage the game will continue with probability p.

Dramatic effect! In the repeated PD, one recovers efficiency exactly as in the infinitely repeated game.

Even if the game is finite with probability 1.

Less intelligent players

Less intelligent? Rock bottom, as the TFT machine in KMRW.

Important class of games: evolutionary games.

Intelligence of players in these games? ZERO!

Dynamics? We'll see. Something works, but we shall need a re-interpretation.

Basic facts

In evolutionary games, strategies are hard-wired. Genetically encoded.

So, players don't "choose" a strategy!

But, then, (is this game theory, and) where is the dynamics?

Dynamics is not at the individual level. It is the population that changes ("learns", "evolves").

Better: it is the distribution of "players = strategies" in the population that changes.

Let's start again

What deserves our attention?

A strategy that resists against invasion of mutant strategies (one at the time, in a small number).

As we did for NE, we don't describe (at first) the dynamics meant by the word "resists".

We try first to "crystallize" an idea of a "rest point". Or a "stable strategy".

ESS

Given a symmetric game in strategic form $((X, Y, f, g)$, with $X = Y = A$, $f(x, y) = g(y, x) = b(x, y))$, a strategy $x^* \in X$ is an evolutionary stable strategy (ESS) if:

for every $x \in X$ different from x^*, there exists $\bar{\varepsilon} > 0$ s.t. the following condition is true for all $\varepsilon > 0$ s.t. $\varepsilon < \bar{\varepsilon}$:

$$(1-\varepsilon)f(x^*, x^*) + \varepsilon f(x^*, x) > (1-\varepsilon)f(x, x^*) + \varepsilon f(x, x)$$

ESS

Equivalent formulation (to see that being a couple of ESS is a condition slightly stronger than Nash equilibrium):

$f(x^*, x^*) \geq f(x, x^*)$ for all $x \in X$

and

$[f(x^*, x^*) = f(x, x^*) \Rightarrow f(x^*, x) > f(x, x)]$ for all $x \in X$, $x \neq x^*$

Remark If $\bar{x}$ is and ESS, then $(\bar{x}, \bar{x})$ is a Nash equilibrium. The converse is not true.

Replicator dynamics

Which is the (a) dynamics behind?

Taylor and Jonker (1978) introduced a dynamics (difference and differential version), based on the effects that strategies have on the fitness of individuals.

What is “fitness”? Essentially is the (expected) number of descendants from that individual.

Connection between ESS and stable points for the replicator dynamics. No details on that. It works fine for “interior” ESS (and interesting connections with NE, as can be reasonably predicted).

In the middle?

In the middle? Not so easy

Nice papers.

Evolution of conventions (Peyton Young). Bargaining via alternating offers (Rubinstein (and others)).

But it is difficult to have a model that really incorporates the intermediate intelligence which is so typical of humans.